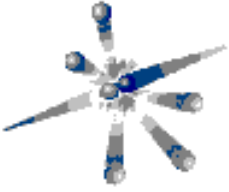


The Electromagnetic Cascade Shower

*A Tutorial Utilizing 3D Color Graphics
(on the Web)*



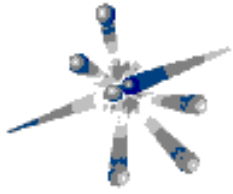
S. H. Rokni and W. R. Nelson
Stanford Linear Accelerator Center
August 2, 2001



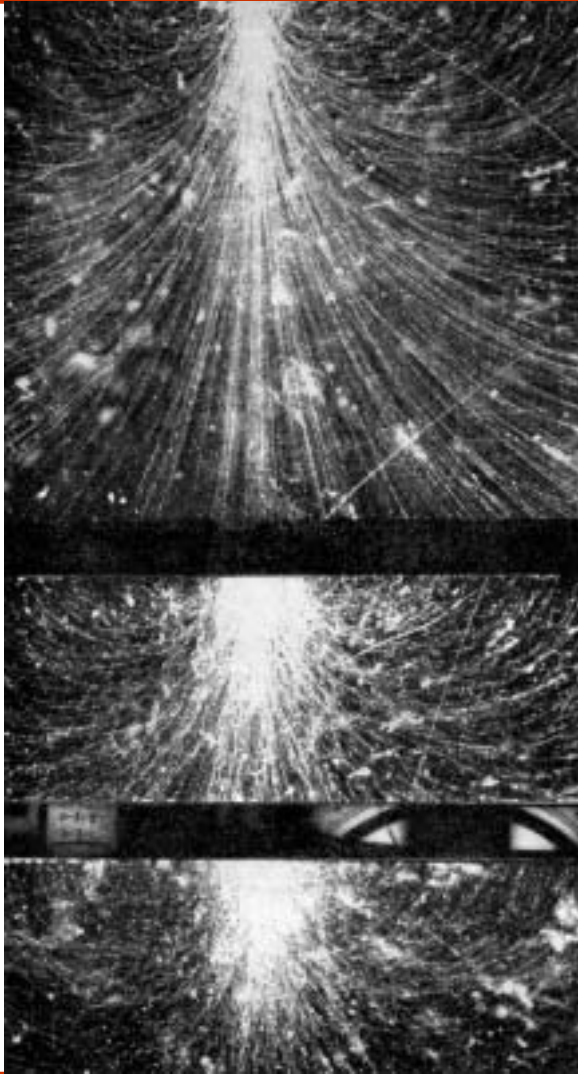
Radiation in General

- Radiation is generally classified as *directly* or *indirectly* ionizing
- Examples of *directly* ionizing radiation include:
 - Electrons
 - Protons
 - Muons
 - Alpha particles
- Examples of *indirectly* ionizing radiation include:
 - Photons
 - Neutrons
- This lecture will concentrate on the interplay of high-energy electrons and photons ...called the

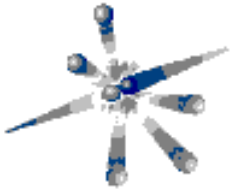
electromagnetic cascade shower



Cloud Chamber Photo – circa 1950's

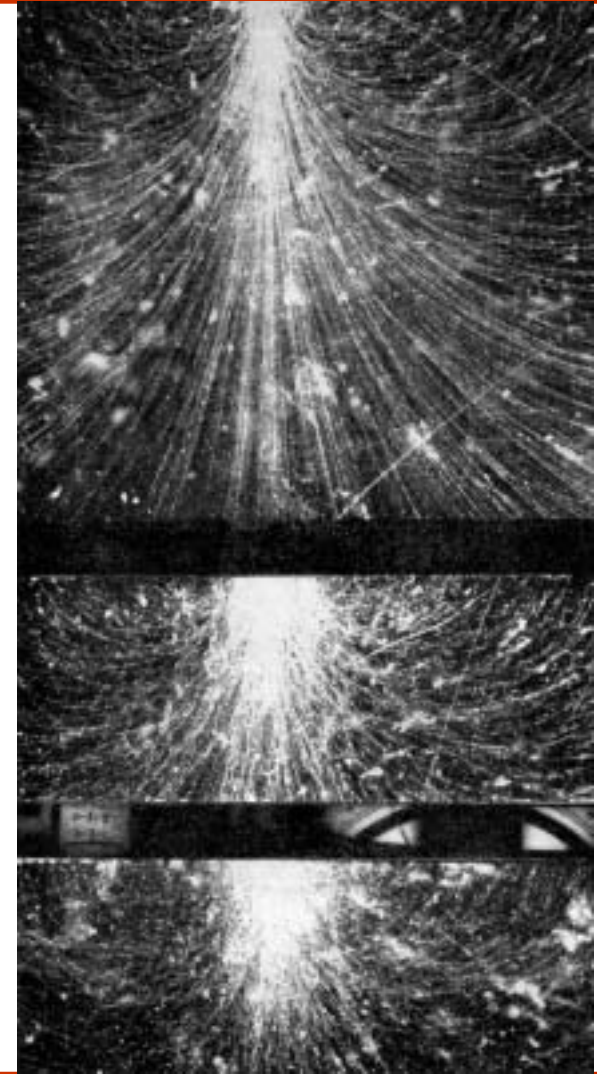


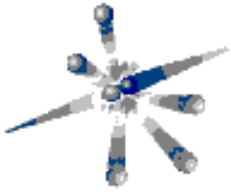
- This is an example of a **shower** produced in a cloud chamber by a cosmic-ray electron (from R. B. Leighton, Principles of Modern Physics (McGraw-Hill, 1959))
- A **single**, very high-energy electron enters the cloud chamber from the top, where it interacts within in a sheet of Pb (not shown)
- The **charged particles** are rendered **visible** by the cloud chamber, but neutral particles (e.g., photons) cannot be seen
- An external **magnetic field** (0.75 kG) has been applied (vertical to the viewing plane) to bend the charged particles
- Two additional **Pb sheets** in the chamber cause shower regeneration to occur



What can we learn from this photo?

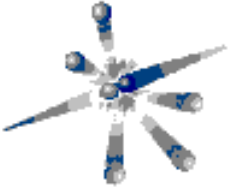
- Starting with just one particle, many more are produced...hence, the name **shower**
- Particles seem to be produced in “pairs” that are oppositely charged
- The shower is easily **regenerated** in the **forward direction** by placing high-density material in its path
- Regeneration may be caused by **neutral particles** – i.e., the charged particles seem to get swept out !
- The energies of the particles are decreasing with shower depth
- Indeed, if the magnetic field were not applied, the shower would be very forwardly directed





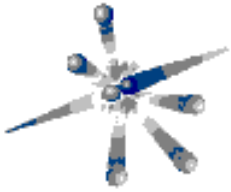
General Description of a Shower

- The EM shower occurs whenever an electron or photon creates **many more** electrons and photons
- Essentially
 - Electrons scatter and emit x-rays (photons, **bremsstrahlung**)
 - Photons materialize into “**pairs**” of electrons (e^+ and e^-)
- As the process develops the incident energy gets shared by more and more particles of **lower energy**
- As a function of depth into the target the number of particles
 - Rises to a maximum
 - And then falls off exponentially
- But remember, other physics processes can (and will) happen



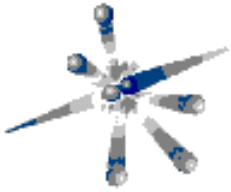
Purpose of this lecture

- To better understand exactly what is taking place in such pictures
- To learn about the individual **physics** mechanisms associated with EM cascade showers
- To put it all together – called **electron-photon transport**
- To tell you about the **transport tools** available
- To point out problems that showers can create...but also tell you about some of their benefits



First of all... a transport tool

- We will make use of a Monte Carlo simulation tool called the [EGS code](#)
- EGS was developed at SLAC in the late 1970's
- Similar to methods used in [Geant4](#) (EGS was the *electromagnetic engine* in the early versions of Geant)
- We will run [EGS on the Web](#) in order to produce color-graphic displays of particle tracks as they interact with matter

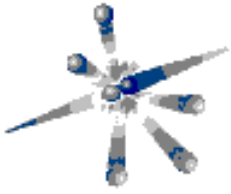


Photon Interactions

(Fano's classification scheme of the 50's)

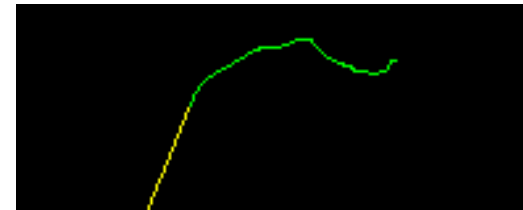
WITH \ TYPE	ABSORPTION	ELASTIC SCATTERING	INELASTIC SCATTERING
Atomic Electrons	Photoelectric Effect	Rayleigh Scattering	Compton Scattering
Nucleons	Photonuclear Reactions	Elastic Nuclear Scattering	Nuclear Resonance Scattering
Electric Field of Surrounding Charged Particles	Pair Production a) Field of Nucleus b) Field of Electron	Delbruck Scattering	
Mesons	Photomeson Production		

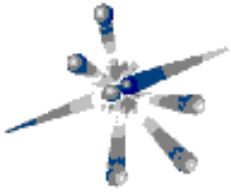
Key: **red** = major process
 blue = minor process
 black = negligible process



Photoelectric Effect

- Process occurs between **photon** and **atom** (not with electron)
- The PE interaction dominates at **low-energies** (< 100 keV)
- There's a **threshold** (or “binding”) energy, B
- Jump discontinuities (“**edges**”) occur in the cross section, corresponding to the atomic shell that is involved
- 80% of the PE interactions involve the K-shell of the atom
- Following a PE interaction the following takes place:
 - Photoelectron is ejected with kinetic energy, $E_{pe} = E_{\gamma} - B$
 - Outer-shell electrons then fill the inner shells (de-excitation)
 - Followed by emission of fluorescent x-rays, Auger electrons, or both
- Typical **EGS tracks** for photoelectric interactions will look like:





Compton Scattering

- Incident photon is scattered by a loosely bound **outer-shell** electron
- Compton dominates in the **mid-energy** range: 100 keV to 10 MeV in Pb (or 100 MeV in water)
- Assuming the electron is “free”, **two-body kinematics** leads to

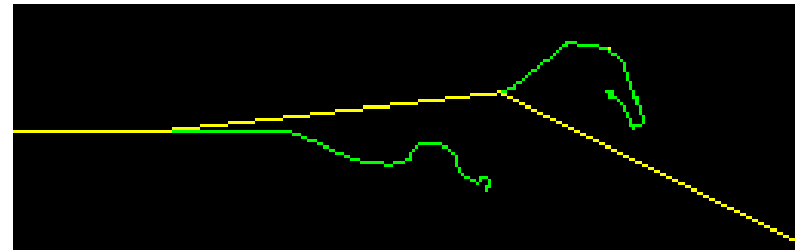
$$E' = E \left[1 + E(1 - \cos \theta) / mc^2 \right]^{-1}$$

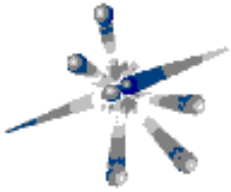
where E, E' = incident, scattered photon energies

θ = angle of scattered photon

mc^2 = rest mass energy of the electron

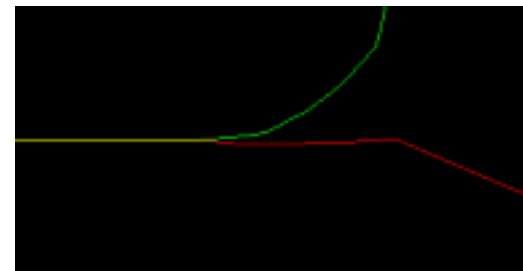
- There is an azimuthal symmetry about the direction of motion
- Typical **EGS tracks** for Compton scatterings will look like:

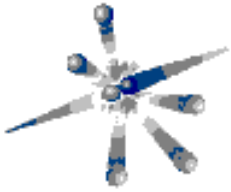




Pair Production

- Process by which photon energy **materializes** into an electron and a positron pair
- Usually occurs in the field of a **nucleus**
- Threshold energy is 1.022 MeV ($= 2mc^2$)
- Pair production is the dominant way photons interact above 10 MeV in Pb (or 100 MeV in water)
- Above these energies the pair production cross section slowly rises with energy
- Pair production and bremsstrahlung are essentially the same process...[they control the EM shower](#)
- Typical **EGS tracks** for pair production interactions look like:





Interaction Probabilities for Photons

- The *mass attenuation coefficient* is a measure of the total interaction cross section (or probability)

$$\mu = \tau + \sigma + \sigma_R + \kappa$$

where

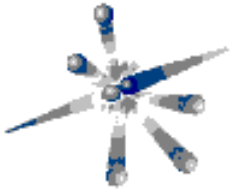
τ = photoelectric effect

σ = Compton scattering

σ_R = Rayleigh scattering

κ = pair production

[Note: Although Rayleigh scattering is usually a minor process, it is included in the above formula because it can be important in some special cases...and it is usually comes tabulated with the other components]



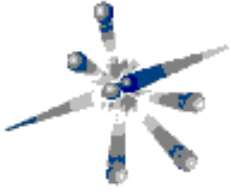
A few words about cross sections

- Generally, cross sections are expressed in units of cm^2/atom or $\text{cm}^2/\text{electron}$
- Alternate forms include:

$$\sigma(\text{cm}^2/\text{g}) = \sigma(\text{cm}^2/\text{atom}) \times 6.023 \times 10^{23} (\text{atoms/mol}) / A (\text{g/mol})$$

$$\sigma(\text{cm}^{-1}) = \sigma(\text{cm}^2/\text{g}) \rho (\text{g/cm}^3) = 1/\lambda (\text{cm})$$

- λ is commonly referred to as a *mean-free path*
- It is the average distance to the next interaction in a material

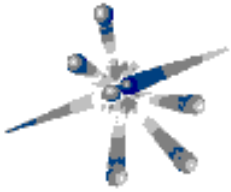


Interaction Probability

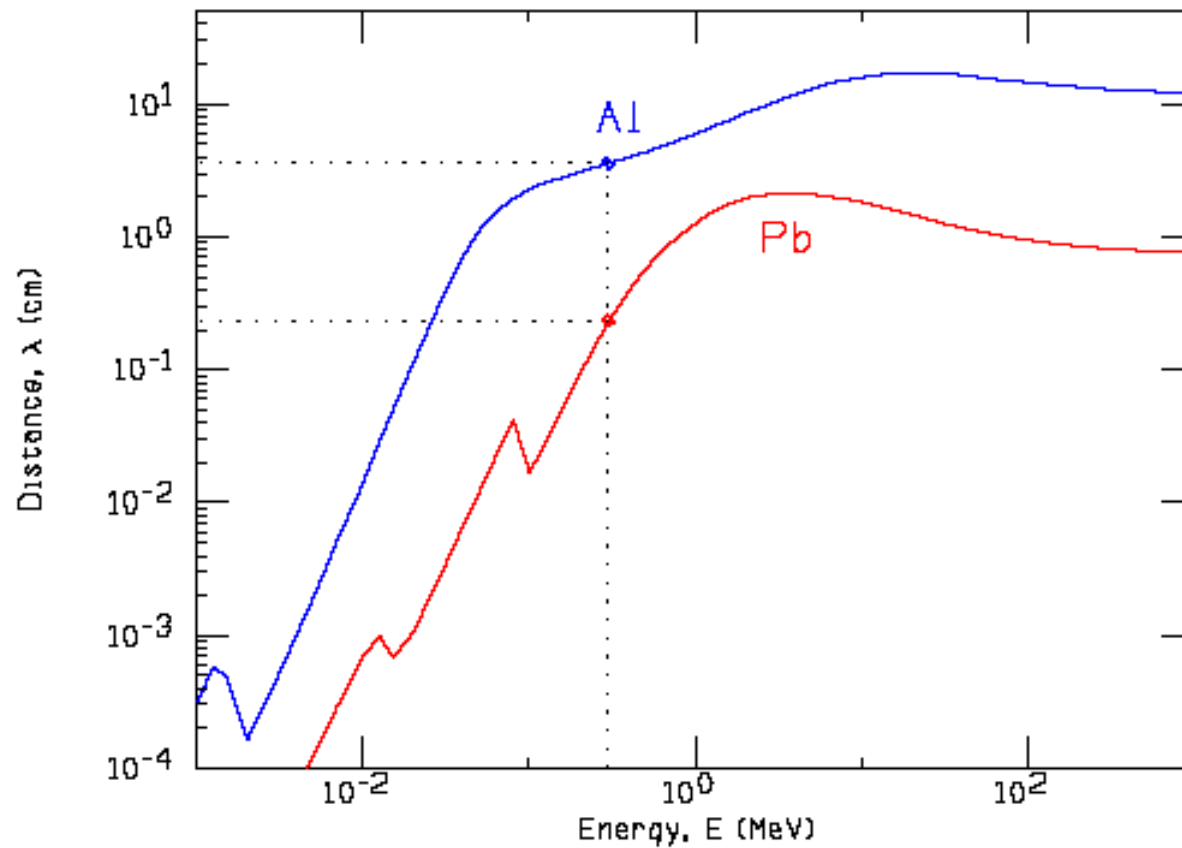
- One can use λ to determine the *probability* that an interaction will occur within a slab of material having a thickness $t(\text{cm})$:

$$P_{\text{int}} = 1 - \exp(-t/\lambda)$$

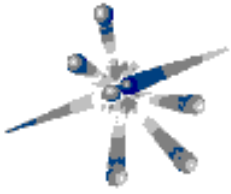
- Note the limits of $P_{\text{int}} = 0$ and 1 correspond to values of $t = 0$ and ∞ , respectively



Using the Mean Free Path: $\lambda = 1/\mu$

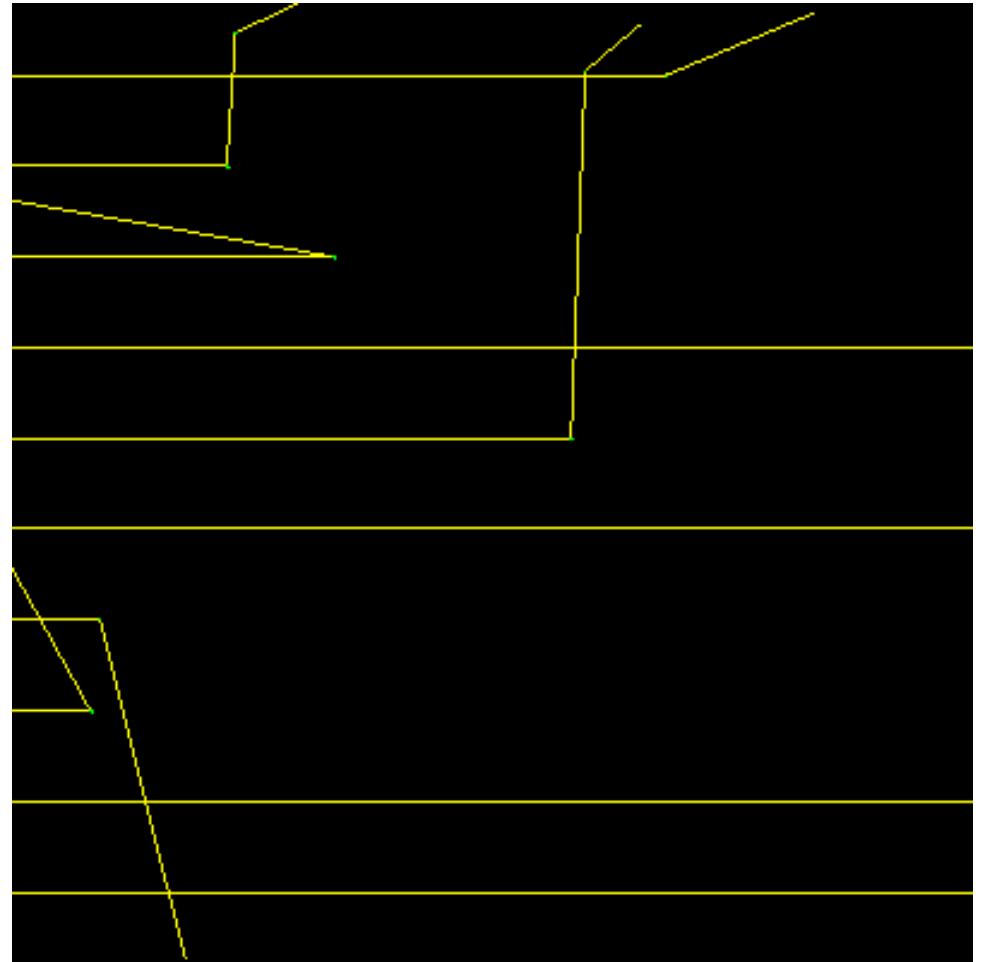


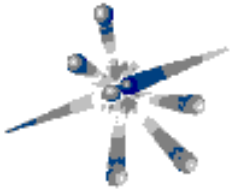
- Consider a **300 keV** photon incident on a **2-cm** slab of either **Al** or **Pb**
- From the plot, the average distance for an interaction to occur is
 - **Al: 3.6 cm**
 - **Pb: 0.23 cm**
- Using the P_{int} formula, the probability that an interaction will occur within the 2-cm slab is
 - **Al: 0.43**
 - **Pb: 1.0**



EGS Simulation: 300 keV photons/2-cm Al

- The photon energy is **300 keV**
- The **Al** slab thickness is **2 cm**
- We have calculated that the interaction probability to be **0.43**
- This means that if we throw ten photons into the slab from the left, on the *average* **4.3** will interact and **5.7** will penetrate
- But in the EGS simulation **6** interact and **4** penetrate !!!
 - What went wrong?
 - Doesn't our theory work?
 - Is EGS making an error?
- Let's have EGS tell us →→→

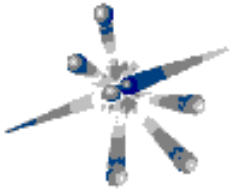




300 keV/2-cm Al

Results for 10 runs with 10 Photons Each

Run Number	Number Interact	Number Penetrate
1	6	4
2	5	5
3	6	4
4	5	5
5	3	7
6	5	5
7	5	5
8	5	5
9	4	6
10	3	7
Total	47	53
Fraction	0.47 ± 0.05	0.53 ± 0.05
Prediction	0.43	0.57



What kind of interactions were they?

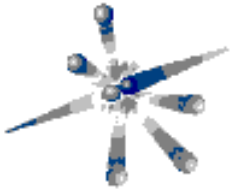
- We have learned about the most dominant photon interactions
 - Photoelectric effect
 - Compton scattering
 - Pair production
- Partial probabilities (called **branching ratios**) are given by

τ/μ = fraction due to photoelectric

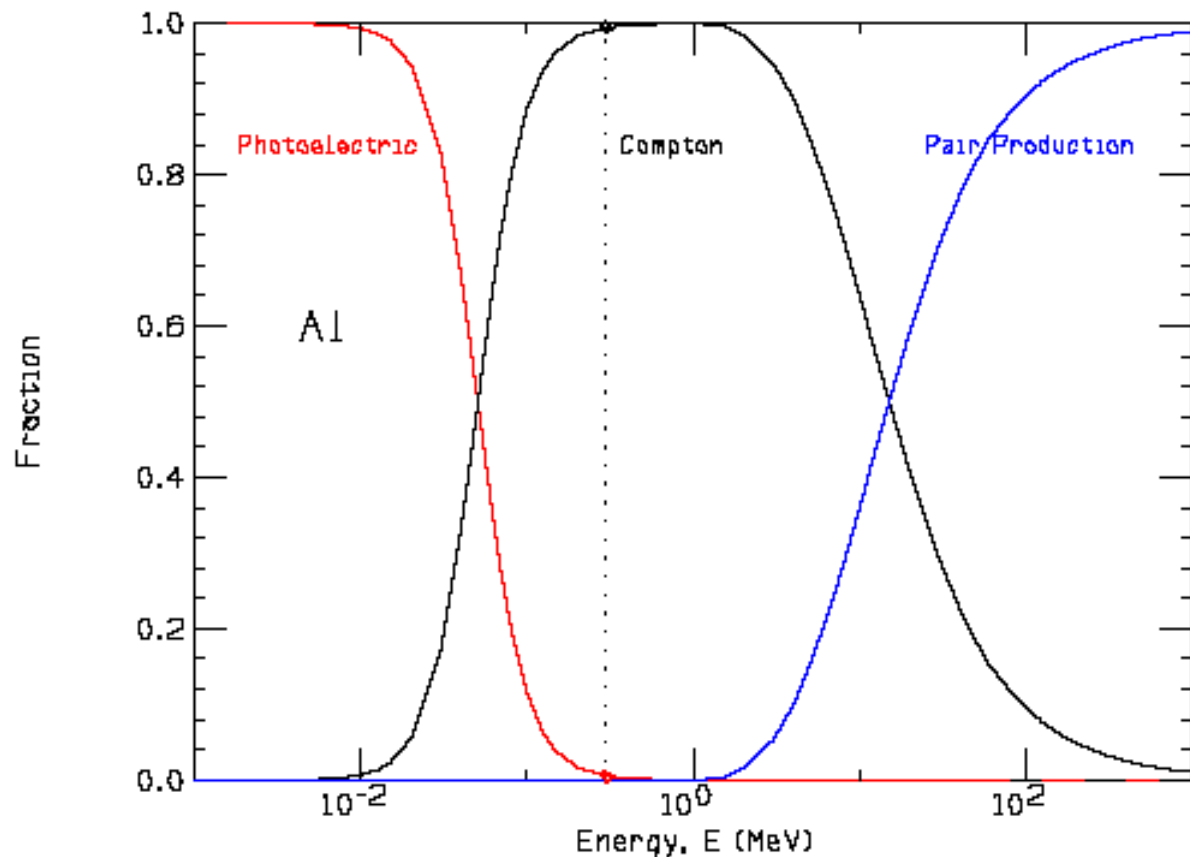
σ/μ = fraction due to Compton

κ/μ = fraction due to pair production

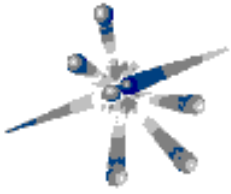
where $\mu = \tau + \sigma + \sigma_R + \kappa$



Branching Ratios for Al at 300 keV

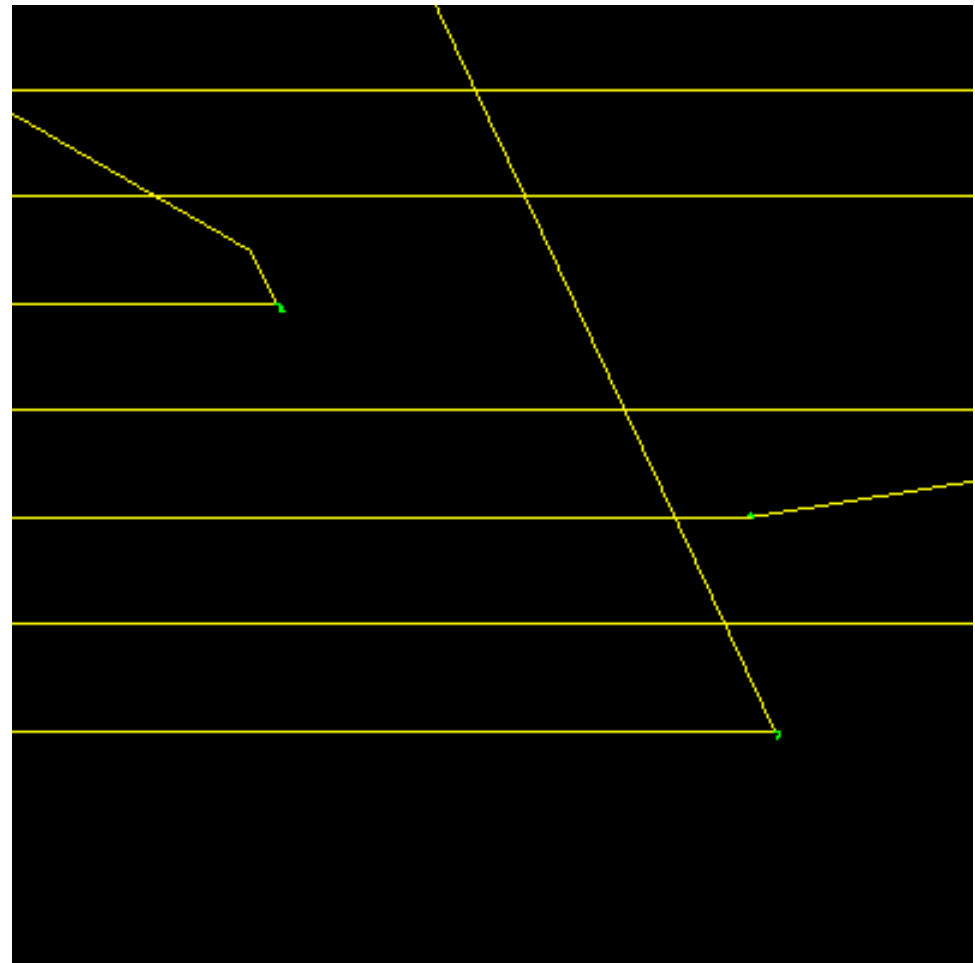
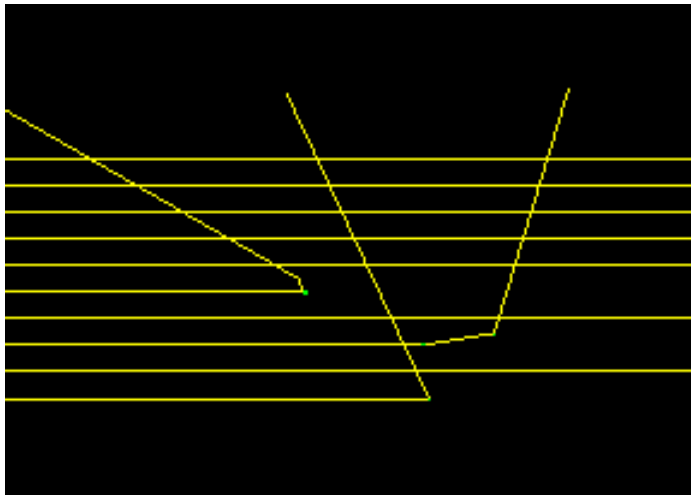


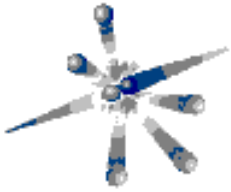
- The PE effect is a low-energy phenomena
- Pair production dominates at high energies (and can't even occur below 1.022 MeV)
- And Compton scattering operates in between
- For our case of **300 keV**, the dotted line tells us that we should expect Compton to occur almost 100% of the time
- **What does EGS say about this? → → →**



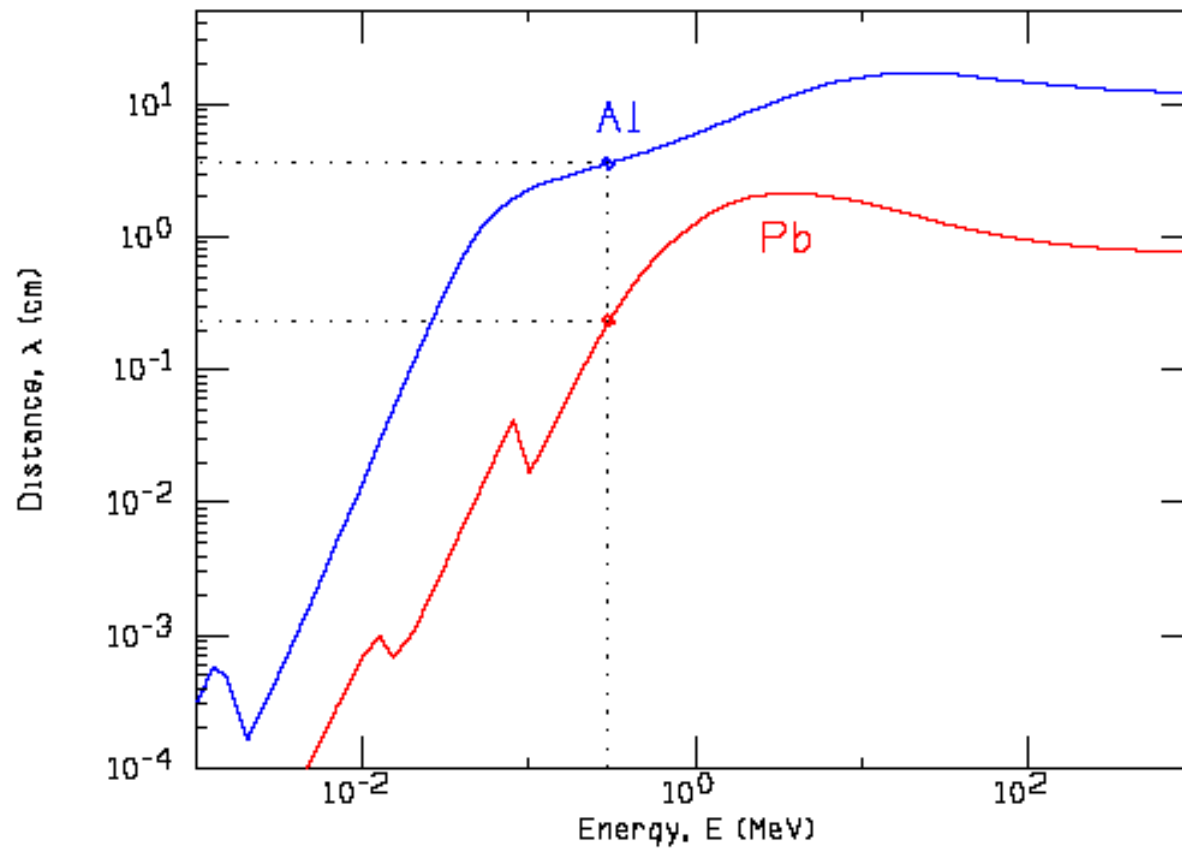
EGS Simulation: 300 keV photons/2-cm Al

- Larger image on the right is a portion of the smaller one below, zoomed by a factor of 4
- Note that there's a (green) Compton electron at the vertex of each scatter

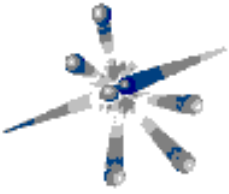




Returning to the mfp curve

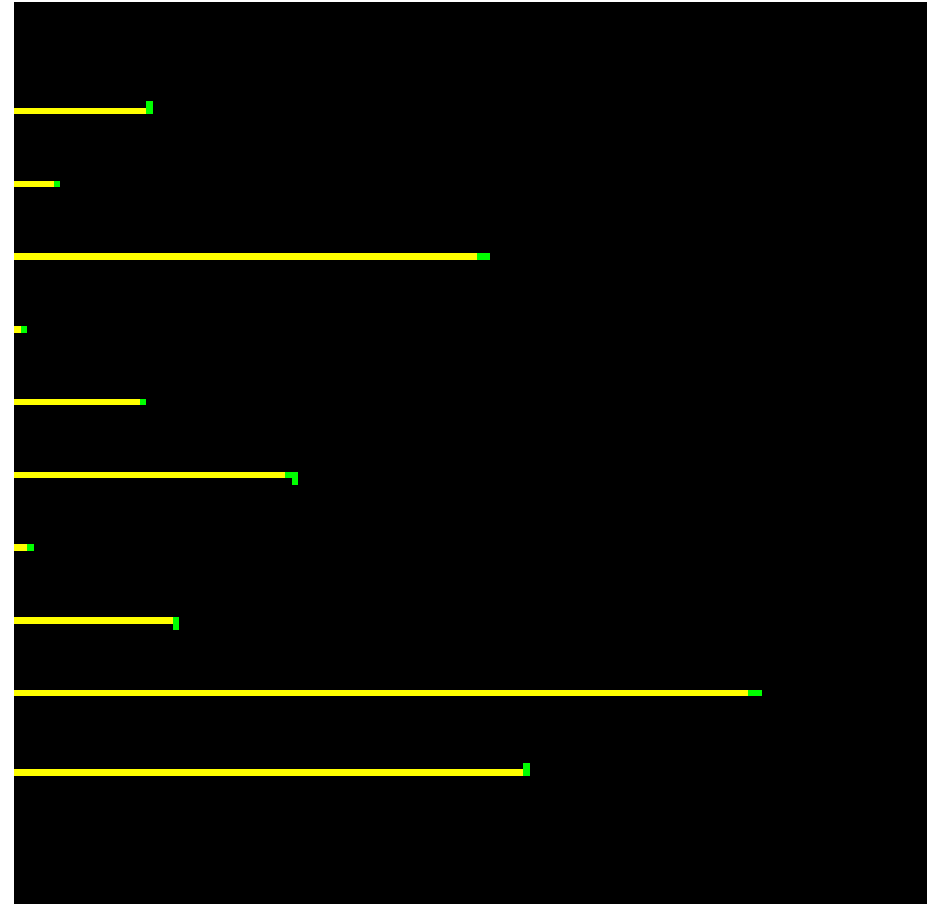


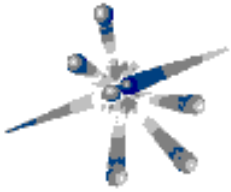
- Consider a **300 keV** photon incident on a **2-cm** slab of either Al or **Pb**
- From the plot, the average distance for an interaction to occur is
 - Al: 3.6 cm
 - **Pb: 0.23 cm ←**
- Using the P_{int} formula, the probability that an interaction will occur within the 2-cm slab is
 - Al: 0.43
 - **Pb: 1.0 ←**



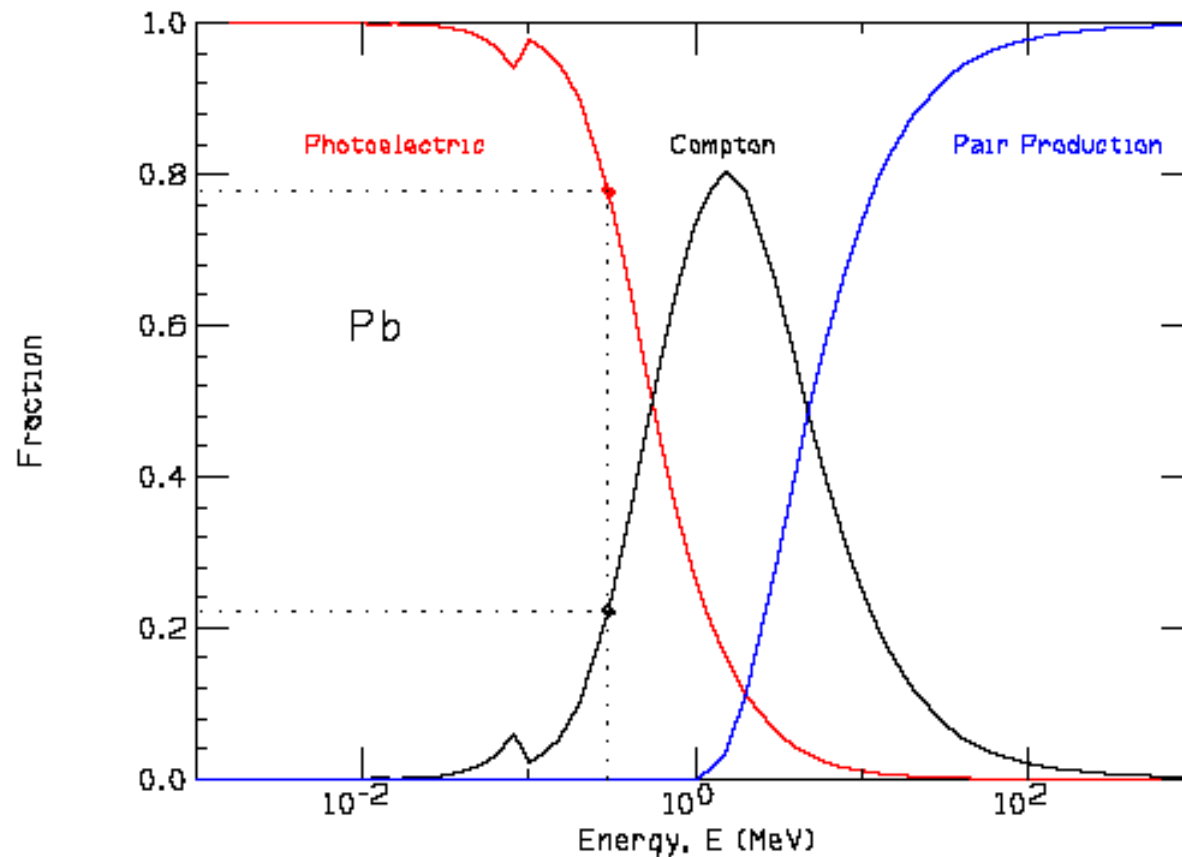
EGS Simulation: 300 keV photons/2-cm Pb

- The photon energy is **300 keV**
- The **Pb** slab thickness is **2 cm**
- We have calculated that the interaction probability to be **1.0**
- This means that if we throw ten photons into the slab from the left, on the *average* **10** will interact and **0** will penetrate
- Here is an EGS simulation →→→
- All ten tracks interact and there is a (**green**) photoelectron at the ends of all of them
- *Could we have predicted this?*

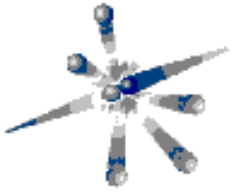




Branching Ratios for Pb at 300 keV



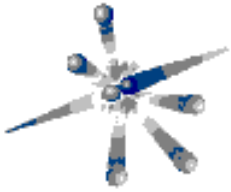
- The curves are slightly different for Pb than for Al, but the same trend applies
- For our case of **300 keV**, the dotted line tells us that we should expect Compton to occur about **21%** of the time
- And **79%** of the time the interactions will be **photo-electric**
- **So what does a higher set of statistics from EGS tell us? → → →**



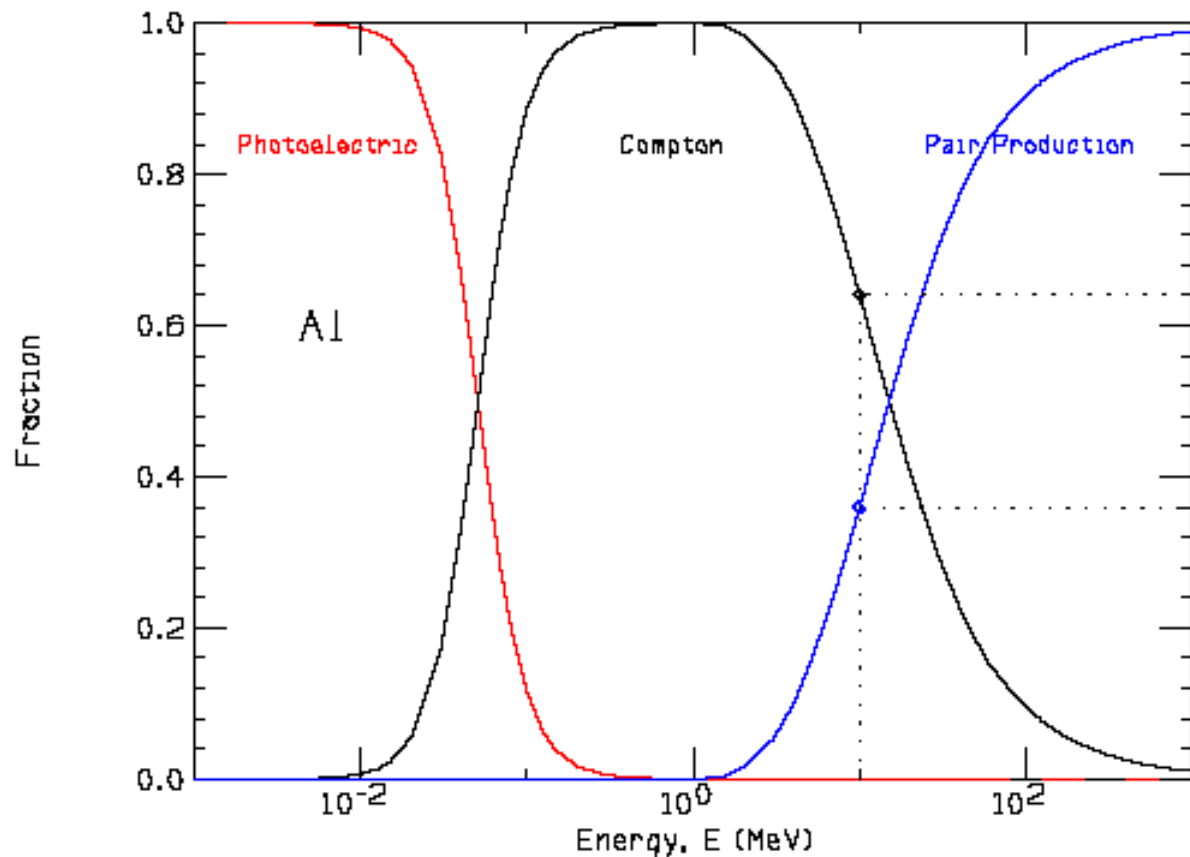
2-cm Pb/300 keV

Results for 10 runs with 10 Photons Each

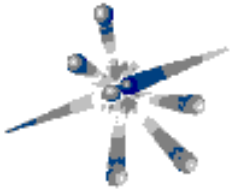
Run Number	Number Photoelectric	Number Compton
1	10	0
2	7	3
3	10	0
4	10	0
5	6	4
6	8	2
7	10	0
8	5	5
9	9	1
10	6	4
Total	81	19
Fraction	0.81 ± 0.08	0.19 ± 0.02
Prediction	0.79	0.21



Branching Ratios for Al at 10 MeV

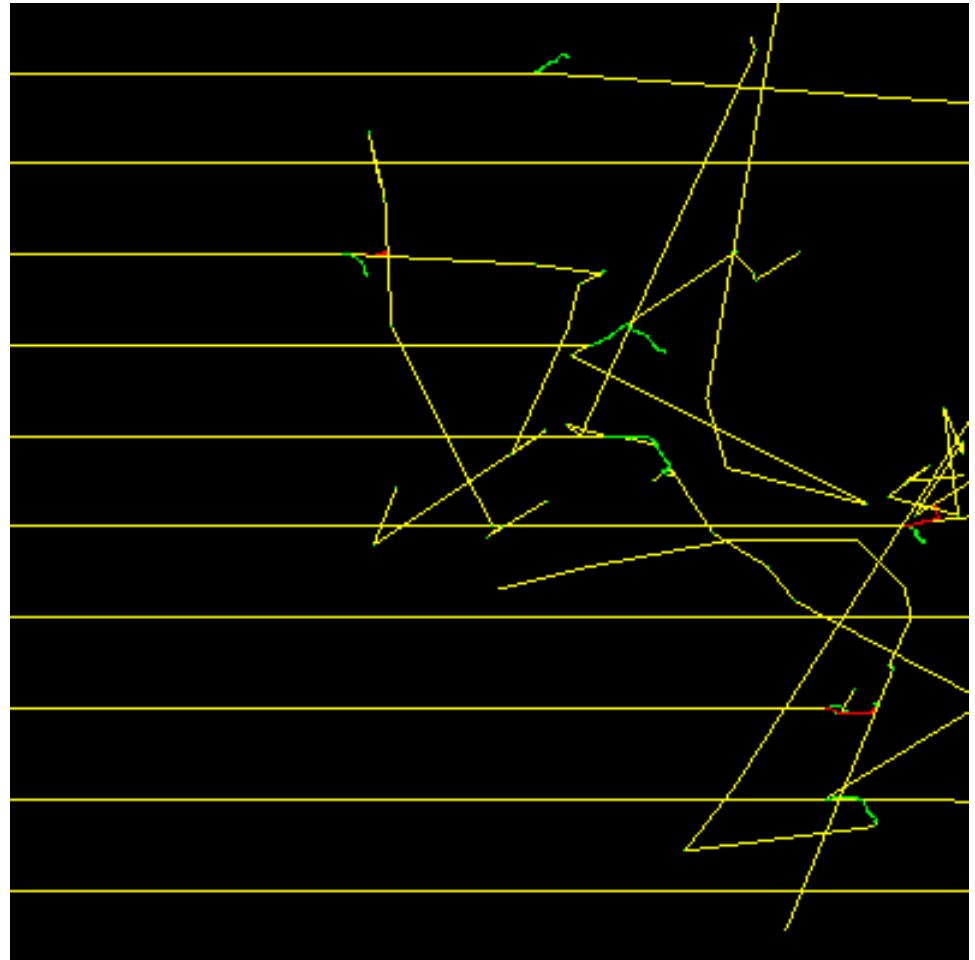


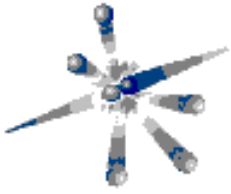
- For the case of **10 MeV**, the dotted lines tell us not to expect any photoelectric
- But we should expect **Compton** to occur about **64%** of the time
- And now we have **pair production** taking place for the first time and we should expect it to happen **36%** of the time
- **And EGS tell us?** → → →



EGS Simulation: 10 MeV photons/20 cm Al

- The photon energy is **10 MeV**
- The **Al** slab thickness is **20 cm** this time (because it helps make the picture prettier)
- The ten events tally up as follows:
 - Three penetrations
 - Four Compton collisions
 - Three pair productions
- Remember -- we are only supposed to tally the **FIRST** interaction of each incident photon, not the the subsequent ones too
- Let's have EGS give us a “statistical” run down →→→

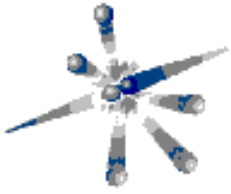




10 MeV/20 cm Al

Results for 10 runs with 10 Photons Each

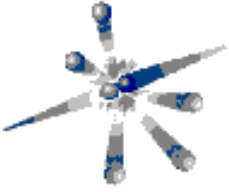
Run Number	Number Penetrate	Number Compton	Number Pair Production
1	3	4	3
2	2	7	1
3	3	1	6
4	1	6	3
5	3	5	2
6	4	3	3
7	3	4	3
8	1	8	1
9	3	6	1
10	1	6	3
Total	24	50	26
Fraction	0.24 ± 0.05	0.66 ± 0.08	0.34 ± 0.04
Prediction	0.29	0.64	0.36



Summary to this point

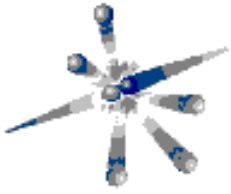
- Photons interact with matter by three dominating mechanisms
- The EGS Monte Carlo code seems to be doing a good job of simulating photon interactions
- What's next in our quest to understand showers?
- Answer:

We need to study the physics of electrons



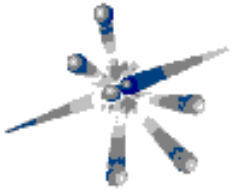
Electron Interactions

- When electrons (+ or –) traverse matter three interaction processes dominate (again)
 - Interactions with atoms as a whole in which the atoms themselves are left in *excited* and (sometimes) ionized states – called *soft* collisions
 - Interactions with orbital electrons in which the collision is *hard* and the **knock-on** electron (or **delta ray**) has a track of its own
 - **Radiative** interactions in which the primary electron is scattered by the field of nuclei with emission of x-rays (called **bremsstrahlung**)



Electron Interactions (cont.)

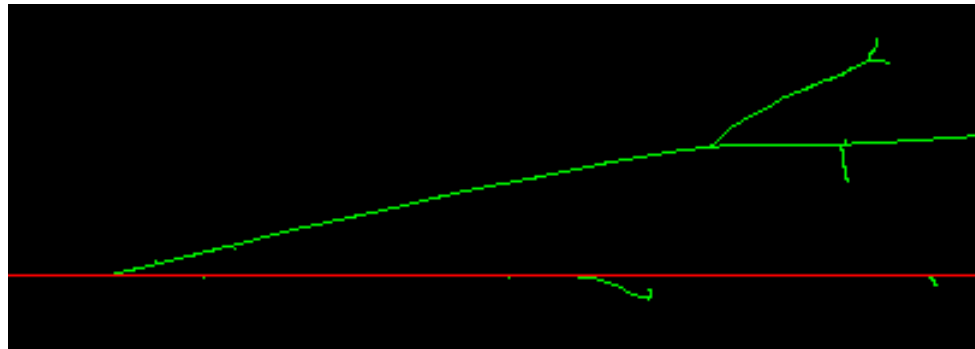
- The **hard** collisions are also classified by the charge of the primary electron and named after the physicist who first determined the cross section for the process
 - Møller: **electron** + electron \rightarrow **electron** + electron
 - Bhabha: **positron** + electron \rightarrow **positron** + electron
- At **low energies** primary electrons basically lose energy by the excitation and ionization of atoms – a combination of both soft and hard collisions – simply referred to as **collision loss**
- Electrons also elastically scatter due to the Coulomb field of the nucleus – very many small elastic scatterings add together and lead to what is called **multiple scattering**



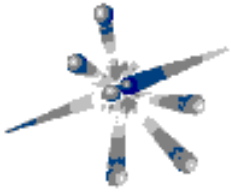
Hard Collisions in EGS

Møller vs Bhabha

- Typical EGS-generated tracks for both Møller and Bhabha interactions are shown in the following picture

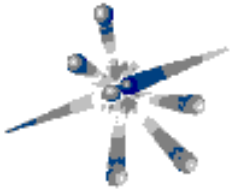


- Moving from left to right, a red positron makes a Bhabha interaction with an atomic electron
- The electron receives enough energy to escape from the atom and make a track of its own – called a delta ray
- The green delta ray moves upward and to the right where it makes a large Møller collision (the V structure)
- Additional Bhabha and Møller interactions are also seen in this picture



Radiative Interactions

- Radiative interactions are inelastic scatters in which x-rays are produced when an electron *decelerates* under the influence of the electric field of the nucleus – hence, the name bremsstrahlung (German for “braking radiation”)
- Whereas ordinary *collision losses* (soft + hard) occur at all energies, the radiative-loss mechanism becomes relatively more and more important as the energy increases
- EM showers are essentially the result of two high-energy processes that feed one another:
 - X-rays produced by electrons (+ or –)
 - Pairs produced by photons
- We can predict at which energy to expect showering to occur by means of the stopping power ...to be discussed next



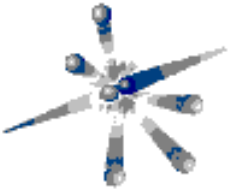
Stopping Powers

(the average rate of energy loss)

- The average rate of energy loss along a charged particle's track is called the **stopping power** and is denoted by dT/dx
- As might be expected, there are two stopping powers:
 - $(dT/dx)_{\text{col}}$ (soft + hard components combined)
 - $(dT/dx)_{\text{rad}}$
- Collision loss vs. radiative loss
 - **Collision** losses occur at all energies but dominate at **low** energies
 - **Radiative** losses kick in at **high** energies and eventually overwhelm the losses due to collisions
- There is a *cross-over* point called the **critical energy** defined by

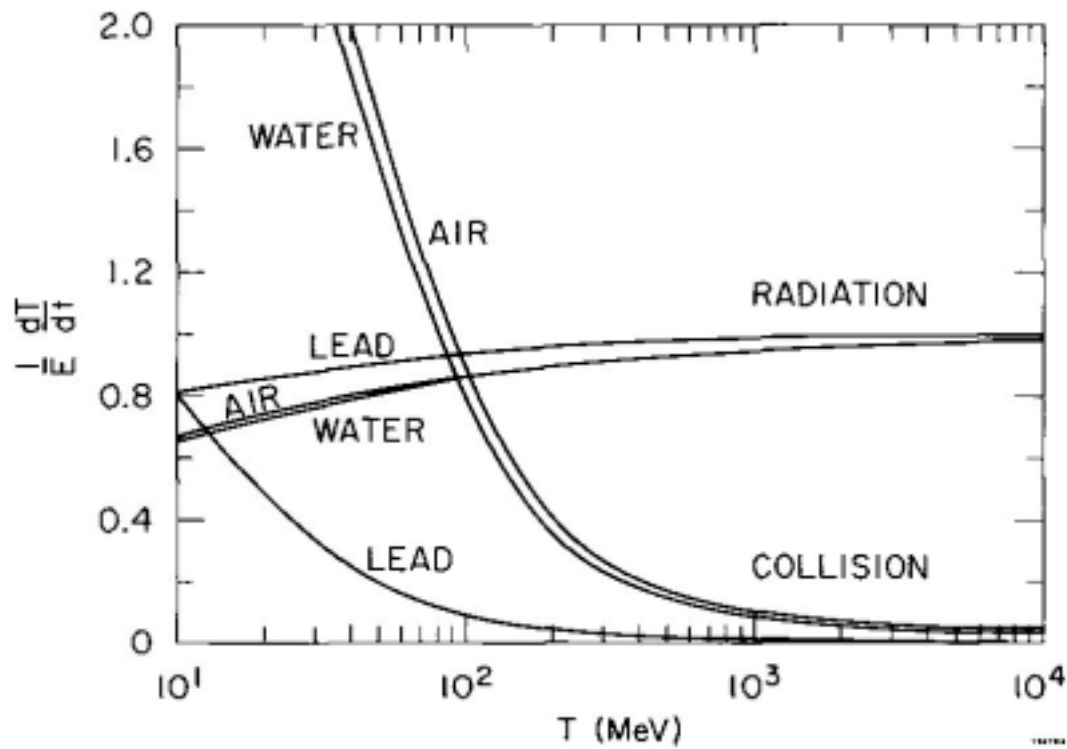
$$(dT/dx)_{\text{col}} = (dT/dx)_{\text{rad}}$$

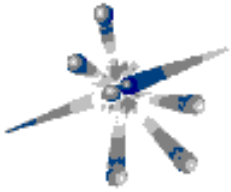
which tells us approximately when to expect the onslaught of showering



Stopping Powers (cont.)

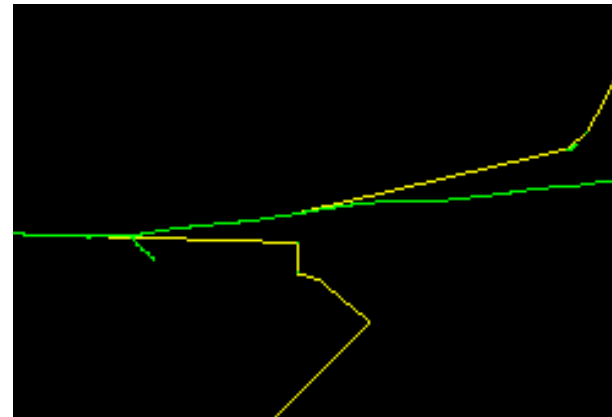
- In this figure the fractional energy loss, dT/Edt , is plotted for both collision and radiative processes, where $t = x/X_0$ (r.l.)
- Cross-overs occur at 10 MeV for Pb and 100 MeV for water
- Also, one easily sees how radiation completely dominates at high energy



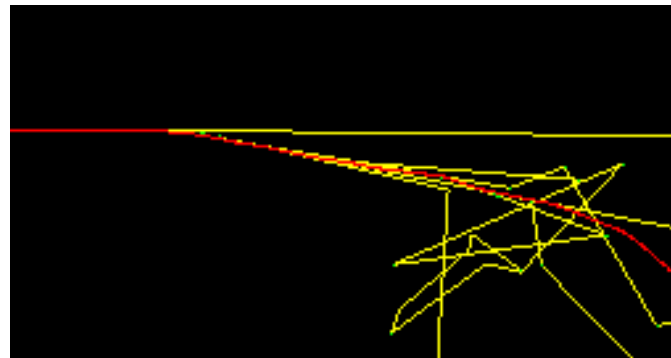


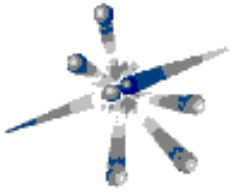
Radiative Collisions as seen by EGS

- Typical EGS-generated tracks showing bremsstrahlung x-rays being produced by electrons



and also by positrons

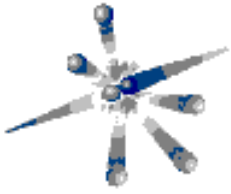




Radiation Lengths

- We have seen that, starting with a single high-energy electron or photon, the number of particles (e^- , e^+ and γ) increases with depth into the medium – the so-called longitudinal development
- A special unit of length, called the **radiation length**, makes it significantly easier to plot these longitudinal shower curves
- That is, we can make showers in different materials (Al, Pb, etc.) scale so that they all fit on the same plot – i.e., they look more or less alike
- The key is knowing that radiative processes dominate at high energies
- We define the radiation length, X_0 , as that distance in which an electron loses $1/e^{\text{th}}$ its energy by emitting x-rays
- Using the bremsstrahlung cross section we can show that

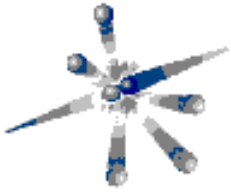
$$\frac{1}{X_0} = 4\alpha Z^2 r_0^2 \ln(183Z^{-1/3}) \text{ cm}^2\text{g}^{-1}$$



Radiation Lengths (cont.)

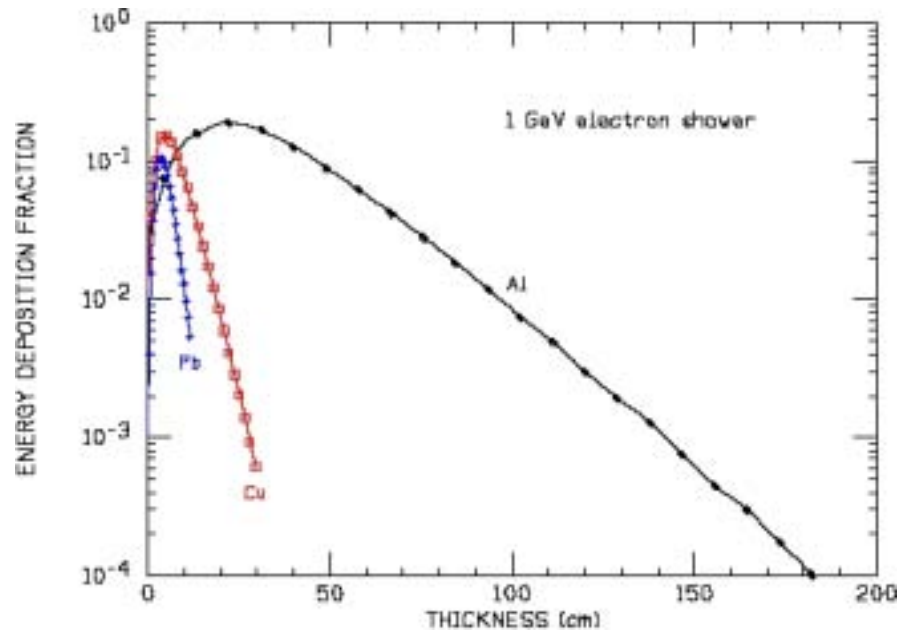
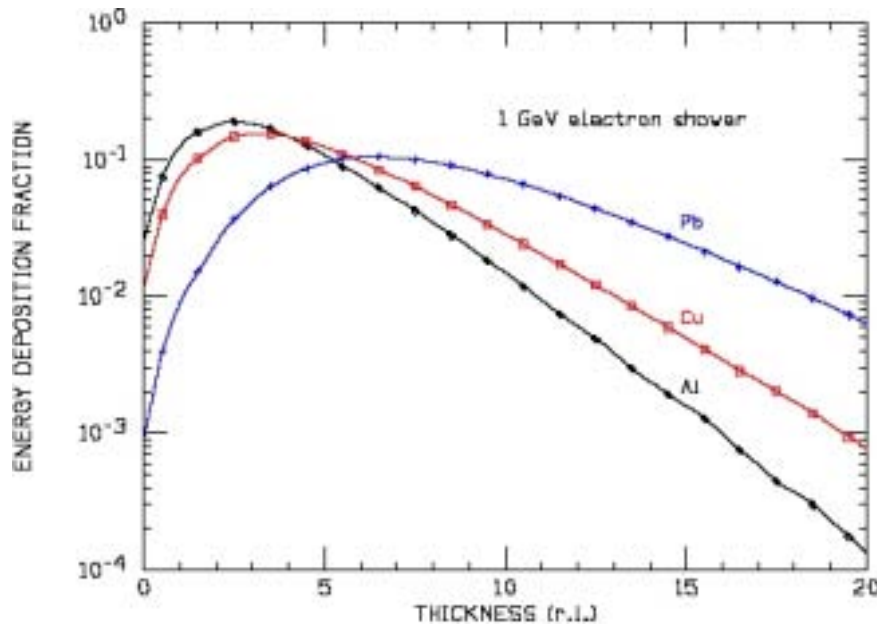
- The phenomena mainly responsible for the development of showers are **radiation** and **pair production** processes
- The theory of pair production also is closely related to that of bremsstrahlung production and consequently the equations describing the **two processes are very similar**
- Therefore, the **natural unit** of thickness in shower theory is the **radiation length**
- Here are a few numbers

Material	X_0 (cm)	E_{crit} (MeV)
Pb	0.56	9.5
Cu	1.4	25
Al	8.9	51
Water	36	92

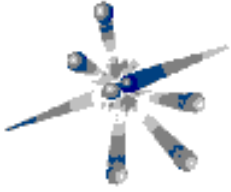


Longitudinal Shower Curves

EGS was used to generate the following figures

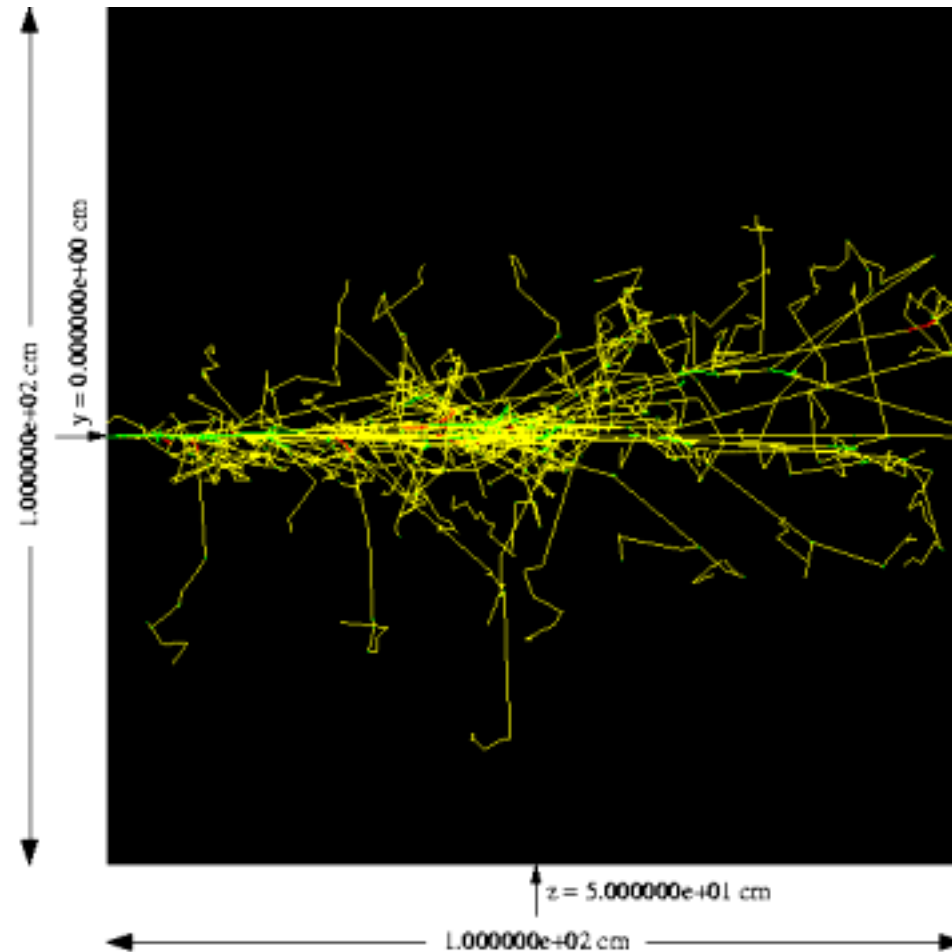


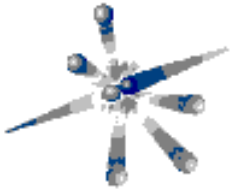
where the benefit of plotting thicknesses in r.l. becomes clear



1 GeV Electron Shower

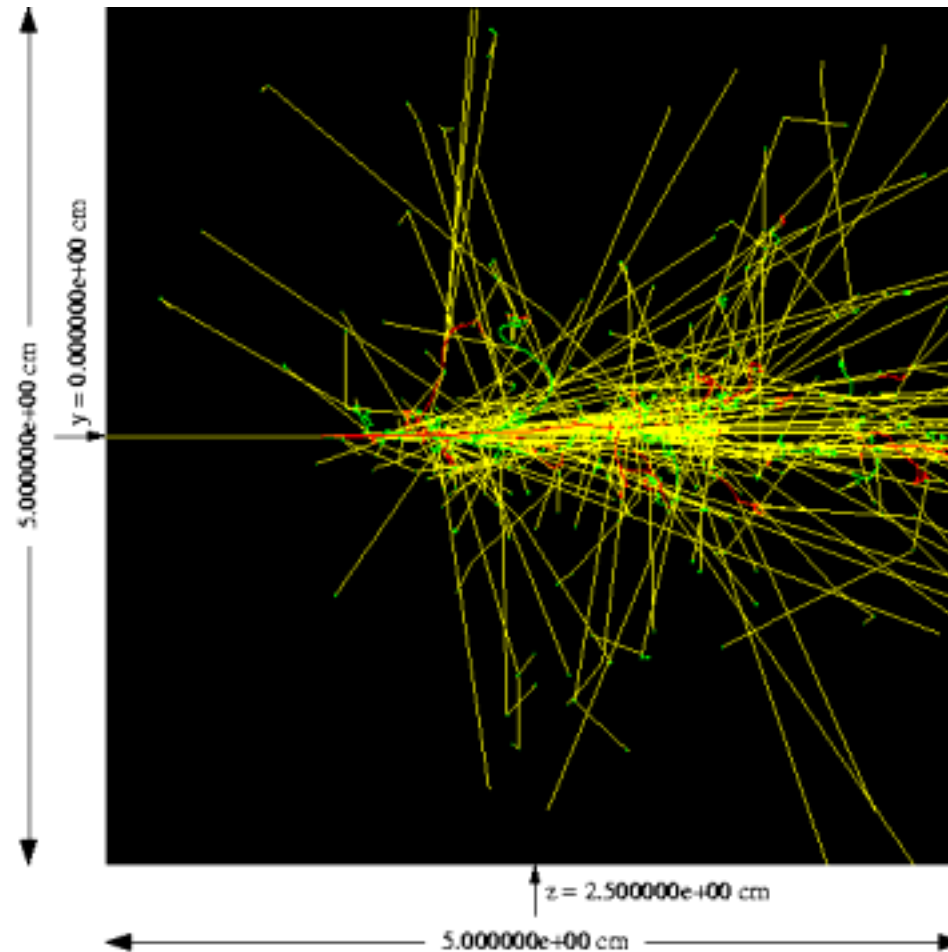
1 GeV electron shower in 10 r.l. of Al (Ecut=0.1, Pcut=0.01 Mev)

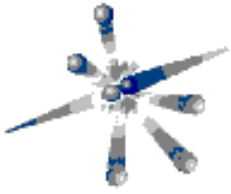




1 GeV Photon Shower

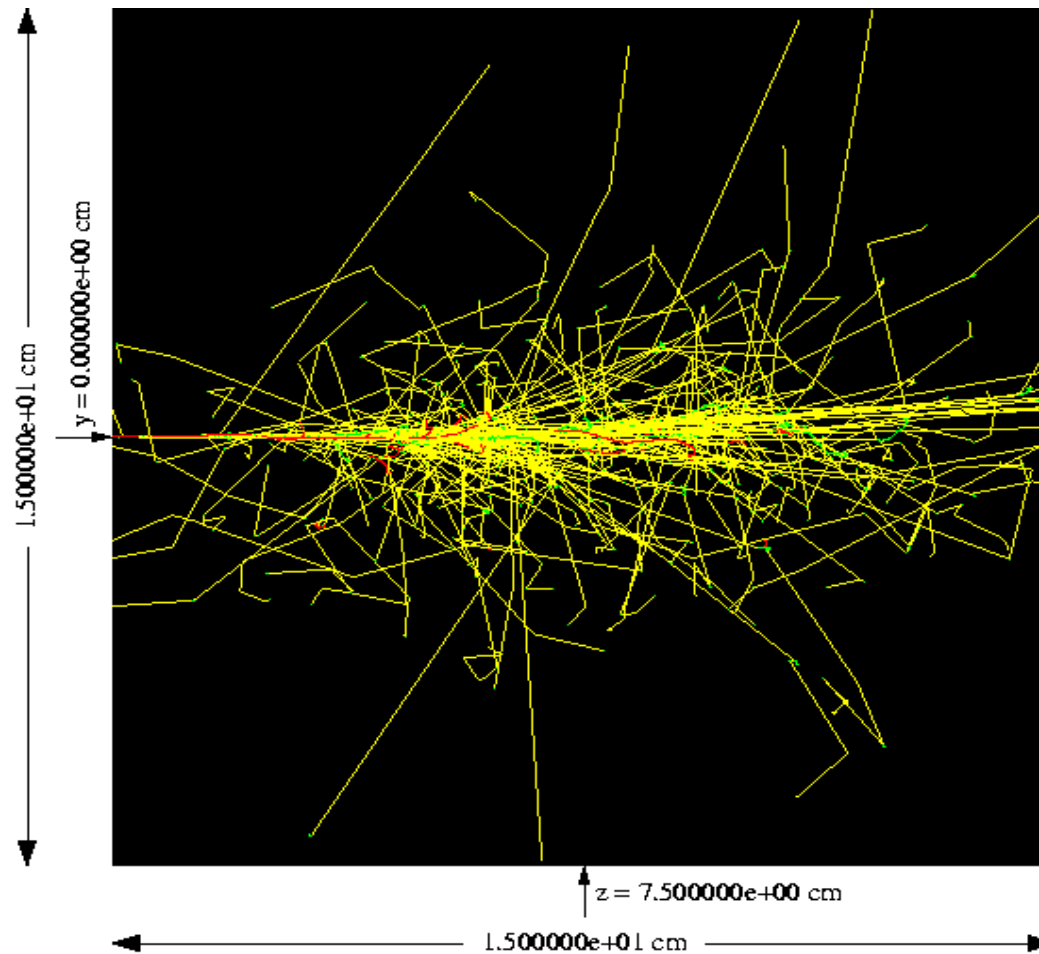
1 GeV photon shower in 10 r.l. of Pb (Ecut=0.1, Pcut=0.01 Mev)

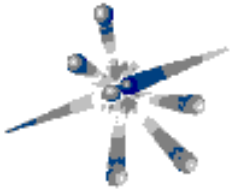




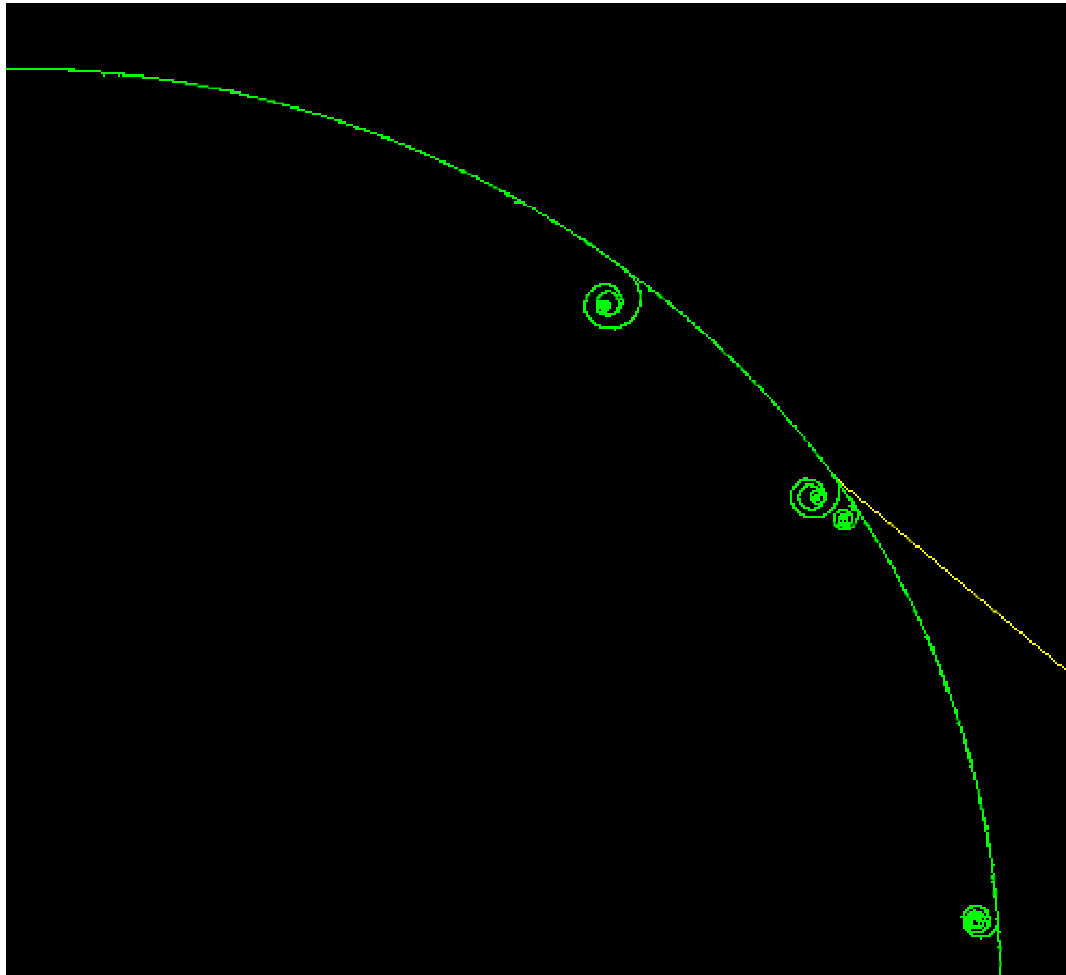
1 GeV Positron Shower

1 GeV positron shower in 10 r.l. of aluminum ($E_{\text{cut}}=0.1$, $P_{\text{cut}}=0.01$ Mev)

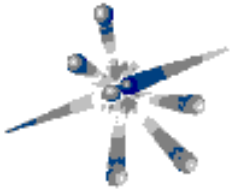




300 MeV Electron in Bubble Chamber

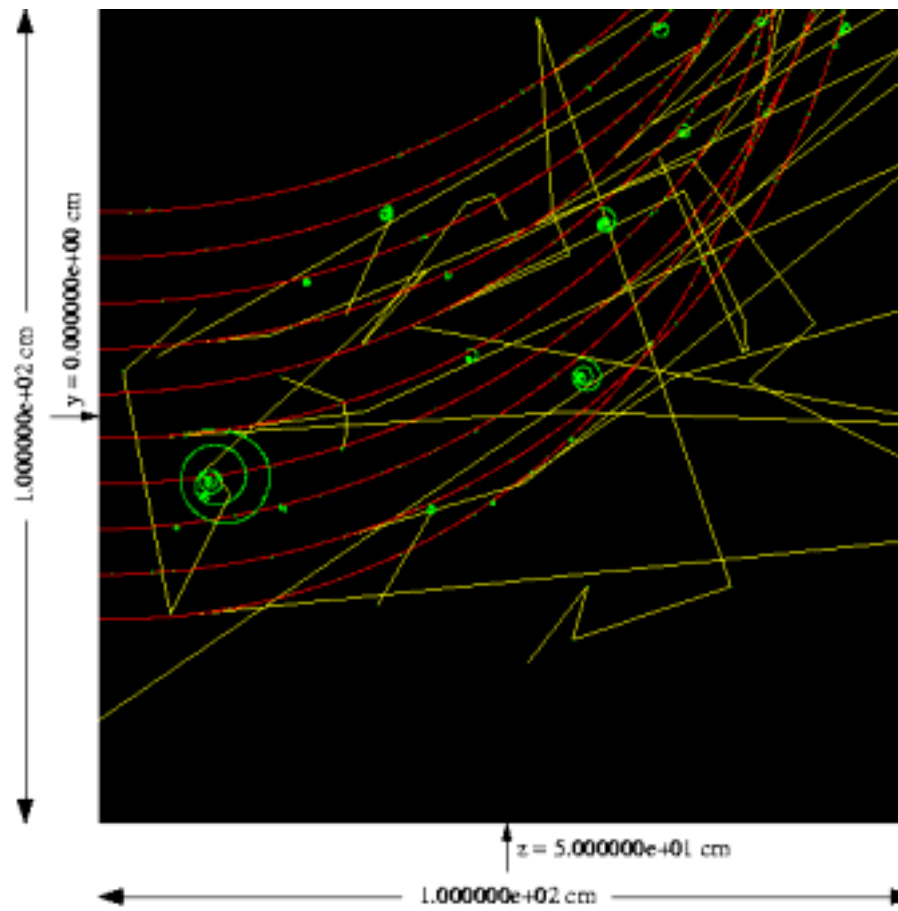


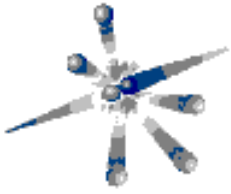
A 300 MeV electron enters a liquid-hydrogen bubble chamber from the top left. As it curves downwards due to a 1 Tesla magnetic field that is being applied, it emits an x-ray (yellow). However, it also makes [Møller interactions](#) (the four little **green** curved tracks).



300 MeV Positrons in Bubble Chamber

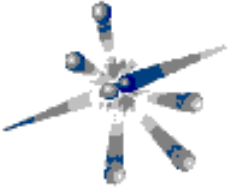
300 MeV positrons 100cm of liquid hydrogen ($E_{\text{cut}}=0.001$, $P_{\text{cut}}=0.001$)
1 Tesla magnetic field applied vertically ($I_{XX}=1$, $J_{XX}=1$) 25 cm spread





Analytic Shower Theory

- Monte Carlo is nice and intuitive, but could we have figured out the shower problem **analytically** – i.e., with a **mathematical model**?
- The answer is: **Yes**, people tried to do it this way in the early days...but *analytic shower theory* turned out to be a formidable problem indeed
- Just to be able to write down a *reasonable* (differential equation) model required severe approximations:
 - Only **one dimensional** – i.e., longitudinal, no angular or radial solutions
 - Only **bremsstrahlung**, **pair production**, **Compton**, and **collision loss** are accounted for
 - Only **high**-energy cross sections used



Analytic Shower Theory (cont.)

- The following set of *diffusion equations* gives you an idea of the math involved:

$$\frac{\partial \pi(E, t)}{\partial t} = -\pi(E, t) \mu_{\pi}(E) + \int_E^{\infty} \pi(E', t) \phi_{\pi\pi}(E', E) dE' + \int_E^{\infty} \gamma(E', t) \phi_{\gamma\pi}(E', E) dE'$$

$$\frac{\partial \gamma(E, t)}{\partial t} = \int_E^{\infty} \pi(E', t) \phi_{\pi\gamma}(E', E) dE' + \int_E^{\infty} \gamma(E', t) \phi_{\gamma\gamma}(E', E) dE' - \gamma(E, t) \mu_{\gamma}(E)$$

where $\pi(E', t)$ = no. of electrons in dE' about E' at t

$\gamma(E', t)$ = no. of photons in dE' about E' at t

$\phi_{\gamma\pi}(E', E)$ = prob. (per r.l.) for photon of energy E'

to produce electron of energy E in dE

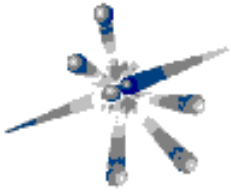
$$= 2\phi_{pair}(E', E) + \phi_{comp}(E', E' - E)$$

$\phi_{\pi\pi}(E', E)$ = prob. (per r.l.) for electron of energy E'

to produce electron of energy E in dE

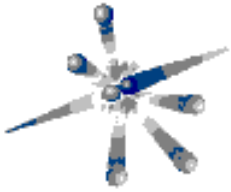
$$= \phi_{rad}(E', E' - E) + \phi_{col}(E', E)$$

$$\mu_{\gamma} = \text{attenuation factor for photons} = \mu_{pair}(E) + \mu_{comp}(E)$$



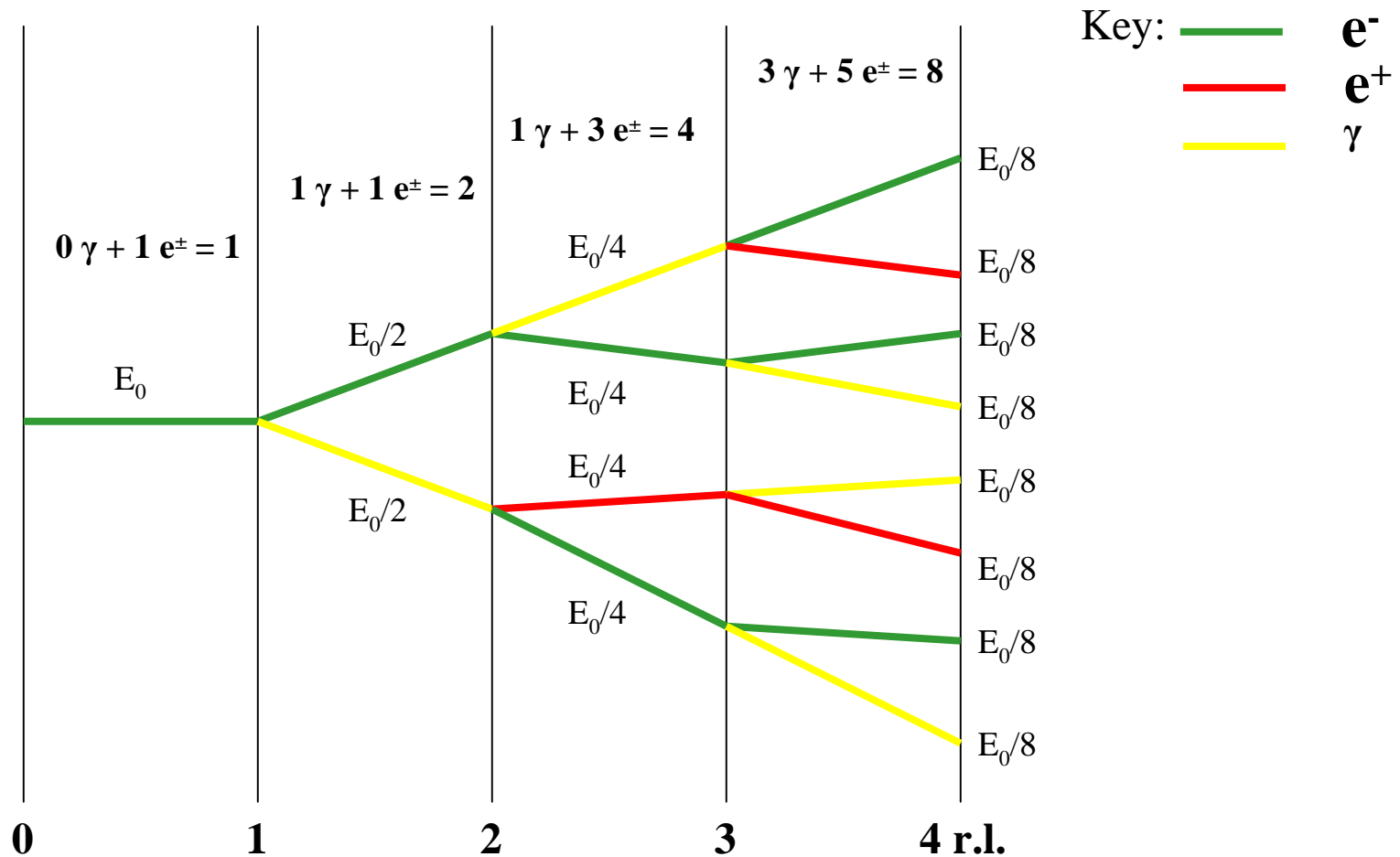
Analytic Shower Theory (cont.)

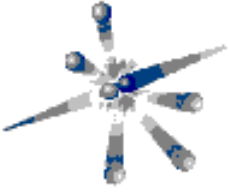
- Pair of coupled integro partial differential equations
- Can be solved using Mellin and Laplace transforms
- Somewhat messy bit of math
- But only applicable to very limited physical situations because
 - No angular or radial information
 - Not all processes are accounted for (e.g., toss out Compton and collision loss to get so-called Approximation A of shower theory)
 - Only high-energy cross section formulae are used
- Nevertheless, this method was better than nothing prior to the invention of Monte Carlo



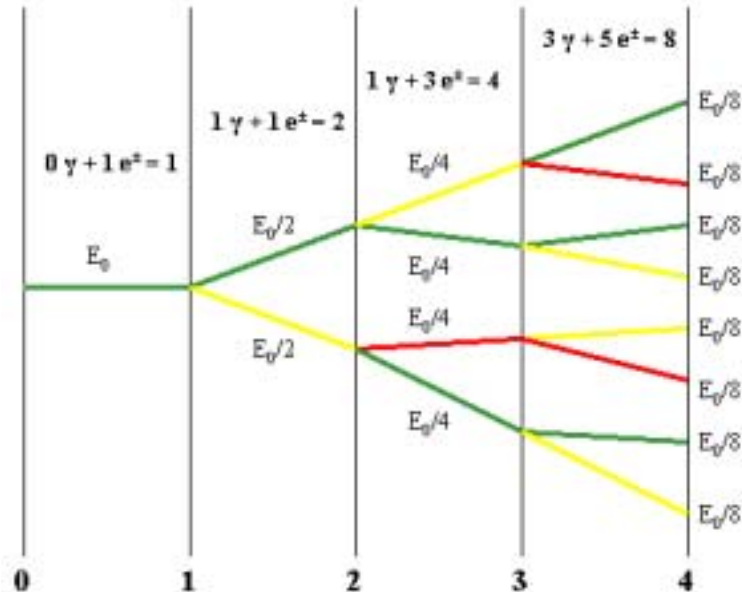
A Very Simple (but instructive) Model

Consider the following sketch:

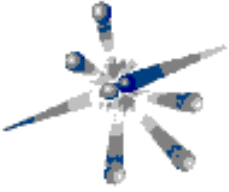




Simple Model... (cont.)



- Assume each e^\pm of energy greater than the critical energy, ε_0 , goes exactly one radiation length ...and then makes a bremsstrahlung photon
- Let the photons make pairs accordingly
- Share the incoming energy equally among the two outgoing particles at each level
- After a thickness of t (r.l.) the total number of particles will be: $N = N_\gamma + N_\pm = 2^t = e^{t \ln 2}$
and the energy of each particle will be: $E = E_0 2^{-t}$



Simple Model... (cont.)

- Thus, the total number of particles with energy **greater than E** will increase exponentially with t from unity (at t=0) to some maximum at

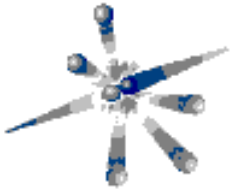
$$t_{\max}(E) = \frac{\ln(E_0/E)}{\ln 2}$$

and then drop abruptly to zero

- The number of these particles at the maximum will be

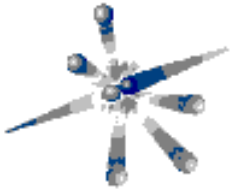
$$N_{\max} = E_0/E$$

- This discontinuous character is a consequence of the assumptions – i.e., the model is **too simple** (one would expect a smooth dependence of N on t)



Simple Model... (cont.)

- Nevertheless...the general features described by this very simple model are expected to be approximately correct (i.e., for energies greater than E):
 - Initially, N_{\pm} or N_{γ} **increases exponentially** with t
 - N_{\pm} or N_{γ} goes through a **maximum** such that t_{\max} is **proportional** to $\ln(E_0/E)$ – i.e., t_{\max} increases slowly with primary energy
 - N_{\pm} (or N_{γ}) is **proportional** to E_0/E
 - The shower curve for the **total** number of particles, irrespective of energy, has a maximum at a thickness of about $\ln(E_0/E_{\text{crit}})$
 - Correspondingly, the **total number** of particles itself is approximately proportional to E_0/E_{crit} (e.g., for a 50 GeV beam in a Pb absorber, $50,000/10=5000$ particles...at shower max)

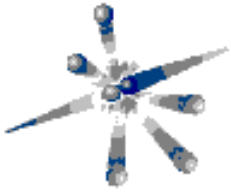


Quick Check of Simple Model

- From the simple theory:

$$t_{\max} = \mathbf{a} \ln(E_0/E) \quad \text{and} \quad N_{\max} = \mathbf{b} (E_0/E)$$

- The **a** and **b** are constants of proportionality (i.e., our **simple theory** is far from being exact...but we are only looking at the systematics of showers!)
- From **analytic shower theory** we have a set of curves (see handout sheet) showing the number of electrons as a function of t (r.l.) for various E_0/E values (plot taken from Rossi's textbook entitled *High-Energy Particles*)
- Using the $E_0/E = 10^4$ curve, we can read off t_{\max} and N_{\max} and solve for **a** and **b**, and then make the table in the next slide

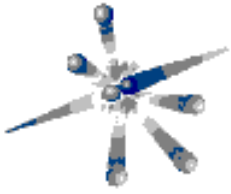


Quick Check of Simple Model (cont.)

- Table with normalization shown in blue

E_0/E	t_{\max}	N_{\max}	$\log_{10} N_{\max}$
10^2	4.0	4.57	0.66
10^3	6.0	45.7	1.66
10^4	8.0	457	2.66
10^6	12.0	45700	4.66

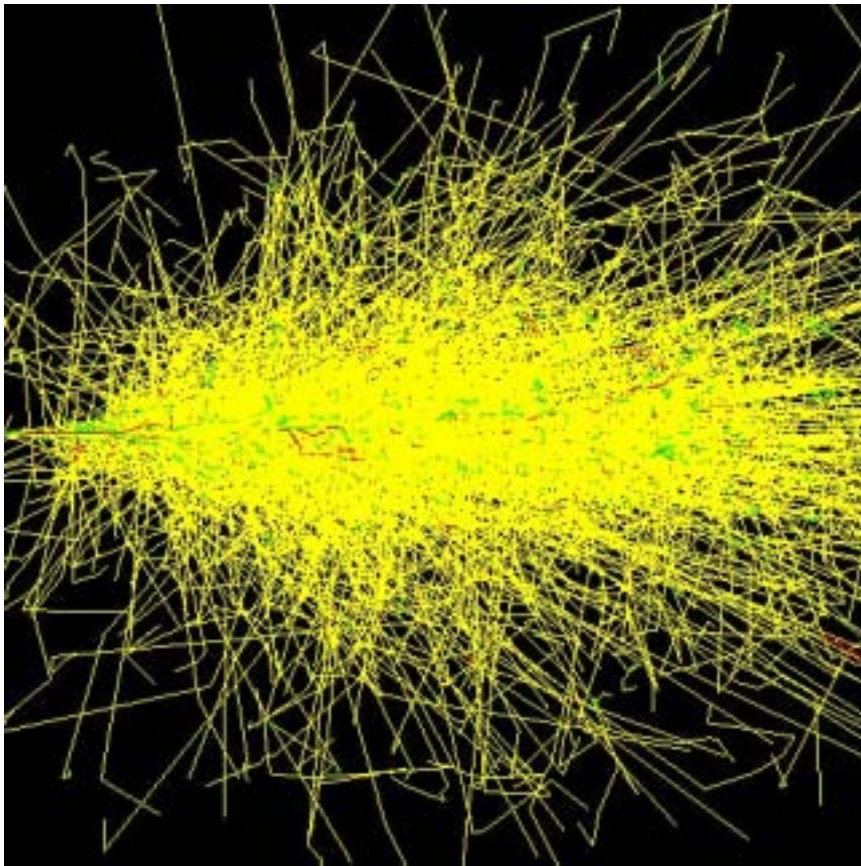
which we want **YOU** to plot on the handout sheet



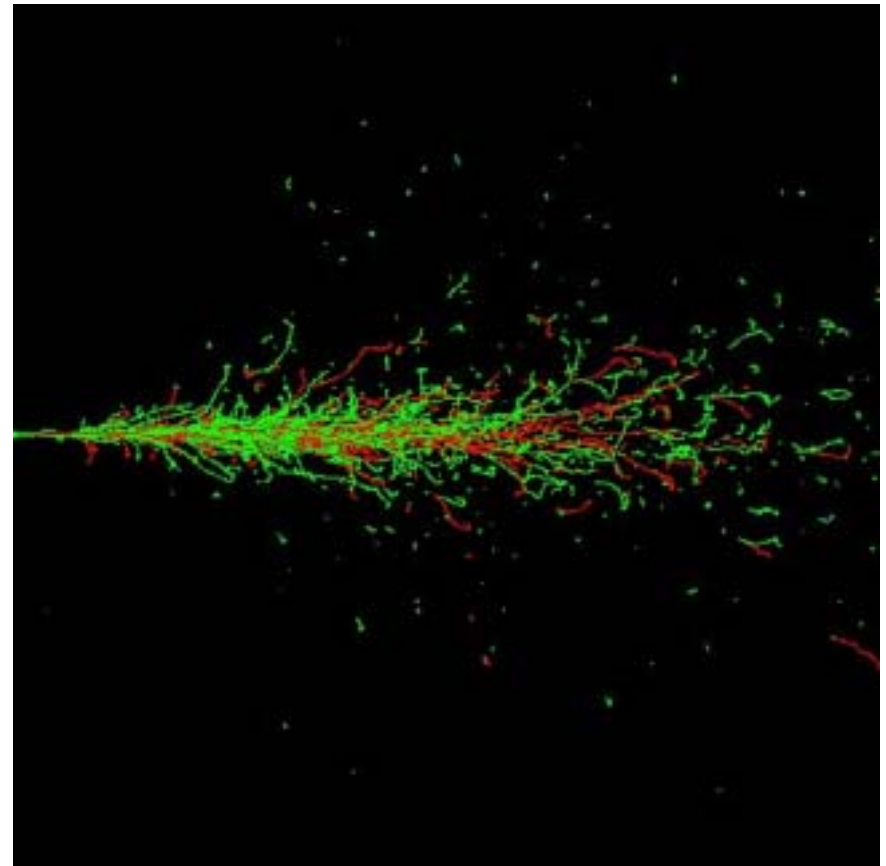
Final EGS Picture...

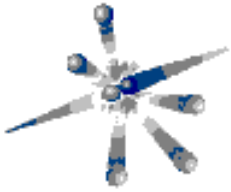
EGS simulation of 1 GeV electron shower in 15 cm of Cu (10 incident)

All particles are shown



Only electrons are shown

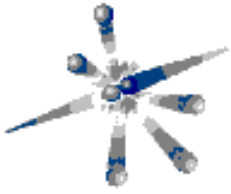




...and a real example

Copper beam stopper (PPS/BCS device) that was destroyed by the beam





Summary

- The physics behind EM cascade showers has been known for a long time
- For some time now we have also been able to quantify the behavior of showers...primarily through Monte Carlo
- Showers can create havoc in high-energy physics
- But showers can also be use to make detectors for high-energy physics
 - calorimeters
 - luminosity monitors
 - Bhabha counters