Accelerating and Focusing Modes in Photonic Bandgap Fibers

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Participant: __________________________
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Research Advisor: __________________________
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1 Abstract

Accelerating and Focusing Modes in Photonic Bandgap Fibers. SARA L. CAMPBELL (Massachusetts Institute of Technology, Cambridge, MA 02142) JAMES E. SPENCER (Stanford Linear Accelerator Center, Menlo Park, CA 94025).

Laser-driven photonic bandgap (PBG) fibers with hollow core defects are one option for future particle accelerators. PBG fiber accelerators use an electromagnetic wave to accelerate charged particles like conventional accelerators do, but they constrain and guide the wave in a new way. Fabrication of PBG fiber accelerators is a challenge because of their small size and high length to diameter aspect ratio. There are several different facilities that can use different technologies to come close to PBG fiber dimensions. To take advantage of well-developed nanofabrication techniques for silicon, several possible silicon PBG fiber accelerating structures were designed and studied. Searches for modes that could accelerate and focus an electron beam were run in the simulation software BandSOLVE and CUDOS. Also, a model of a fiber from Incom was studied further in CUDOS. The simulated field data from CUDOS was then analysed in MATLAB and scaled to find the maximum achievable accelerating gradient. Structures that support viable accelerating modes were indeed found in silicon. The accelerating gradients of all of the structures were found to be one to two orders of magnitude better than what is possible with present technology. Also, an electro-optical focusing mode was found in one of the silicon structures and accurately modeled as an electric sextapole.
2 Introduction

Photonic bandgap (PBG) fiber accelerators are a possible new particle accelerator that could achieve a much higher accelerating gradient than is currently possible. At the Stanford Linear Accelerator Center (SLAC) linear accelerator, electrons are accelerated by microwaves that travel down large metal radio frequency (RF) cavities. PBG fiber accelerator technology seeks to accelerate electrons by laser light that travels down small dielectric PBG fibers, a kind of photonic bandgap crystal. PBG fibers made of both fused silica and silicon were studied, as current fabrication procedures could eventually make the PBG fibers needed for accelerators by either pulling fused silica glass or drilling into silicon wafers.

2.1 Photonic Bandgap Crystals

Photonic crystals, an optical analogy to atomic crystals, are periodic lattices of macroscopic dielectric media. That is, the dielectric constant \( \epsilon(\mathbf{r}) \) varies periodically with the position \( \mathbf{r} \) in the medium. Electromagnetic modes are waves of a particular frequency \( \omega \) that solve Maxwell’s equations,

\[
\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) e^{i\omega t}, \quad \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{i\omega t} \tag{1}
\]

\[
\nabla \times \mathbf{E}(\mathbf{r}, t) + \frac{1}{c} \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} = 0, \quad \nabla \times \mathbf{H}(\mathbf{r}, t) - \frac{1}{\epsilon(\mathbf{r})} \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} = 0. \tag{2}
\]

Combining equations (1) and (2) gives

\[
\nabla \times \left( \frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = (\frac{\omega}{c})^2 \mathbf{H}(\mathbf{r}) \equiv \Theta \mathbf{H}(\mathbf{r}) = (\frac{\omega}{c})^2 \mathbf{H}(\mathbf{r}). \tag{3}
\]

The differential operator \( \Theta \) is Hermitian and thus has real, positive eigenvalues \( (\frac{\omega}{c})^2 \). Electromagnetic modes are eigenvalue solutions to the Hermitian equation[1].

The periodically varying dielectric constant \( \epsilon \) forces conditions that do not allow modes of certain frequencies. These disallowed frequency ranges are called bandgaps. When light of a disallowed frequency does enter the crystal, it is not a mode, rather, its amplitude decays exponentially with \( z \), the longitudinal distance of propagation[1].
2.2 Photonic Bandgap Fiber Accelerators

Photonic bandgap fibers are photonic crystals that are periodic in the transverse \((xy)\) plane and are uniform in the longitudinal direction. The PBG fibers in this study were made of a background dielectric material (silica or silicon) with cylindrical holes arranged in hexagonal rings. Accelerating structures introduce a defect to break the periodicity and shift lattice modes in frequency so that they are actually in the bandgap. In PBG fiber accelerators, the defect is introduced by making the radius of the center hole larger. Figure 1 shows a CAD model of the Lin fiber[2], a photonic bandgap fiber with a central defect. Modes that get shifted into the bandgap are called defect modes, because the electromagnetic waves can only exist in the defect, as they are at frequencies forbidden in the wider periodic lattice.

An electron beam can propagate in the defect and be accelerated by a defect mode confined to the same region. The motion of a charged particle in electric and magnetic fields is described by the Lorentz force equation, where \(\mathbf{F}\) is the force on the particle, \(q\) is the charge of particle, and \(\mathbf{E}\) and \(\mathbf{B}\) are the electric and magnetic fields.

\[
\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \tag{4}
\]

Thus, accelerating modes must have longitudinal electric fields for particle acceleration and transverse magnetic fields to confine the beam. For constant acceleration, an electron in the defect must stay at a peak of the electromagnetic wave accelerating it. Since electrons in particle accelerators approach the speed of light, we must accelerate them with speed of light (SOL) modes[3].

The PBG fiber is made out of dielectric material with higher breakdown fields than metals, so it can support much higher electric fields than an RF cavity without being damaged. Also, modes chosen for acceleration are mostly confined to the hollow defect, so damaging energy does not leak out into the surrounding structure, improving efficiency. The PBG fiber can support a much higher acceleration gradient than a traditional RF cavity, because of its superior structure and higher damage threshold. Another advantage is that PBG fibers have the capability to simultaneously support a variety of modes. Electron beams are traditionally
focused by a series of alternating quadrupole magnets external to the RF cavity. In a PBG fiber we can excite focusing modes at the same time as accelerating modes, so these structures have the potential for new, built-in focusing capabilities.

2.3 Fabrication

Suitable structures for PBG fiber accelerators require holes in dielectrics with diameters on the order of 1 micron, and lengths on the order of 1 centimeter. At such small scales this 1000:1 aspect ratio has not yet been obtained, though some companies and laboratories are beginning to come close. The fiber optics company Incom, Inc.[4] is producing borosilicate microcapillary arrays. Currently, their microcapillary arrays are long enough, but do not have holes of small enough radii and do not have a defect that would cause necessary the defect modes. The Advanced Accelerator Research Department (AARD) at SLAC has been coordinating with Incom, and several new samples from Incom have been obtained. AARD is also collaborating with the Center for Integrated Systems (CIS)[5] at Stanford University. CIS can fabricate photonic bandgap structures that meet accelerator specifications by drilling into thin silicon wafers. These nanofabricated structures can be made small enough and they can be made to have a defect. However, the best option for drilling, a photo-assisted electrochemical etch, only gives a 100:1 aspect ratio.[6] Neither Incom nor CIS can fabricate structures with all of the required dimensions, but they are good possibilities for the near future. This project searches for and studies accelerating and focusing modes that can occur in PBG fibers that could be fabricated at Incom and CIS.

3 Materials and Methods

3.1 BandSOLVE simulations

The simulation programs BandSOLVE[7] and CUDOS MOF Utilities[8] were used to find and characterize modes in PBG fiber structures. BandSOLVE assumes that the fiber is repeated as an infinite periodic lattice and solves the eigenvalue equation (3) to find both bandgaps for the lattice and defect modes in the bandgaps.[9] Any mode can then be ex-
pressed using a plane wave expansion in terms of these eigenmodes. BandSOLVE gives a graph of the $\omega$ vs. $k_z$ dispersion relations for the modes found in the bandgaps, so the approximate laser wavelengths that will excite SOL modes can be selected. BandSOLVE is then used to perform a rough search for defect modes inside the SOL intersection regions. BandSOLVE was used to design and then modify the design for the new silicon PBG fiber. First, lattice dimensions that support many SOL line crossing bandgaps were found. Then, the defect radius ideal for supporting defect modes was determined.

3.2 CUDOS simulations

CUDOS uses the approximate wavelengths of the defect modes found by BandSOLVE to do a finer mode search on the actual finite structure. CUDOS expresses the local electric fields in each of the cylinders as a sum of cylindrical harmonics using a Fourier-Bessel expansion set to have order 5. CUDOS then solves for the coefficients of this expansion by using boundary conditions to couple the fields between the cylinders[10]. Once the silicon PBG fiber was designed in BandSOLVE, searches for accelerating modes and focusing modes in the fiber were run in CUDOS. The approximate defect mode wavelengths from BandSOLVE helped narrow the search. Also in CUDOS, Elliott Johnson’s model of a sample fiber from Incom in silica[11] was scaled down to a size that could be driven by an available 2 micron wavelength Thulium-doped YAG laser (Tm:YAG). After modes were found, the electric and magnetic field data was exported into MATLAB for calculation of the accelerating gradient and multipole strengths. Only the electric and magnetic field magnitudes were exported into MATLAB, so information about the signs of these fields was lost.

CUDOS also solves for the effective index of refraction for the mode in the fiber. The fiber is an arrangement of different dielectric media, so the effective index, or the index of refraction that the mode actually “sees,” depends on the particular mode and the way it propagates. We can understand what the complex component of the effective index means in the following way: Components of an electromagnetic wave traveling in the $z$ direction are proportional to $e^{i(kz-\omega t)}$, where $k$ is the wave number of the wave and $\omega$ is the angular frequency. If the wave is traveling purely in the $z$ direction, then $k$ and $\omega$ are related
by the following dispersion relation $k = \omega / v = \omega n / c$, where $v$ is the speed of the wave, $n = n_{re} + i n_{im}$ is the complex index of refraction of the material, and $c$ is the vacuum speed of light. Combining these expressions gives,

$$e^{i(kz)} = e^{i(\omega (n_{re} + i n_{im}) c)} = e^{\frac{\omega n_{re}}{c}} e^{\frac{-\omega n_{im}}{c}}. \tag{5}$$

From equation (5), we see that $n_{im}$ is a damping term, related to how the magnitude of the wave decreases as it propagates[12]. Accelerating modes must have very low complex indices of refraction, in order to minimize mode leakage and damage to the material. Typical accelerating modes have $n_{im} \approx 10^{-5}$, which means that the amplitude of the fields will fall to $1/e$ of the original field after the wave has traveled for approximately 1 meter.

3.3 MATLAB Analysis

BandSOLVE and CUDOS output the relative field magnitudes. To calculate the actual electric field in the defect and thus the accelerating gradient, the relative distribution must then scaled to the maximum field strengths the accelerating structure can support. A MATLAB[13] script was written to sort through the data, scale it, and plot it. Part of the script locates the maximum electric field in the exported CUDOS data and scales all field values so that the maximum electric field in the structure is set to the breakdown electric field. Though precise damage thresholds in dielectrics such as fused silica and silicon are still unknown, estimates of the breakdown electric fields are obtained from previous work done by the AARD at SLAC[14][15].

For fused silica in air, the damage threshold $D$ of a $\Delta t = 150$ fs pulse of 800 nm radiation has been measured to be 2.03 J/cm$^2$[14]. From this, we can calculate the average Poynting vector $S$, where $E_{rms}$ is the root mean square electric field in the wave and $Z$ is the impedance of the medium,

$$S = \frac{D}{\Delta t} = \frac{E_{rms}^2}{Z} \tag{6}.$$  

Now, we can find $Z$, knowing fused silica’s index of refraction $n = \epsilon_r = 1.46[16]$ at the wavelengths of interest, where $\epsilon_r$ is the relative dielectric constant, and the impedance of
free space $Z_0 = 377$,

$$Z = \sqrt{\frac{\mu}{\epsilon}} \approx \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{Z_0}{\sqrt{n}} = \frac{377}{n} = 258 \Omega. \quad (7)$$

Finally, we find the average electric field $E_{\text{rms}}$ and the peak electric field $E_0$,

$$E_{\text{rms}} = \sqrt{\frac{ZD}{\Delta t}} = 5.91 \times 10^9 \text{ V/m} \quad (8)$$

$$E_0 = \sqrt{2E_{\text{rms}}} = 8.36 \text{ GV/m}. \quad (9)$$

For silicon in air, the damage threshold $D$ of a $\Delta t = .69$ ps pulse of 2100 nm radiation has been measured to be $0.24 \text{ J/cm}^2[15]$. We calculate $E_0$ similarly, noting that at the wavelengths considered the index of refraction of silicon $n = 3.45[16]$,

$$Z = \frac{Z_0}{n} = 109 \Omega \quad (10)$$

$$E_{\text{rms}} = \sqrt{\frac{ZD}{\Delta t}} = 6.41 \times 10^8 \text{ V/m} \quad (11)$$

$$E_0 = \sqrt{2E_{\text{rms}}} = 906 \text{ MV/m}. \quad (12)$$

MATLAB was also used to locate the boundaries between the different materials along the lines on which the fields were plotted. For the focusing modes, MATLAB was used to find polynomial fits for the different field distributions.

4 Results

4.1 Further studies on IncomD fiber

Elliott Johnson’s model of an Incom fiber, called the IncomD fiber, was successfully scaled down for available laser wavelengths. The longitudinal electric field $E_z$, shown in figure 2, was scaled for the material and plotted as a function of both $x$ and $y$ in figures 3 and 4, respectively. The accelerating gradient of this structure is 5.55 GeV/m. Note that the electric
field is constant in the defect where the electrons pass. This is critical for all accelerators, because the electron beam must be accelerated uniformly.

4.2 New silicon structure

4.2.1 Accelerating Modes

The dimensions of the new silicon structures designed are listed in Table 1. The previously studied Lin fiber[2], made out of silica, was used as a reference. When the index of refraction $n$ of a material is increased, the dispersion relations of its bandgaps tend to shift down in frequency, under the speed of light line. This is because, for a given free space wavelength of light $\lambda$, the frequency $\nu = c/n\lambda$, so increasing $n$ effectively reduces the frequency. The lattice radii also determine the scale of the bandgap wavelengths, so they were made larger than the Lin lattice radii, to match the decreased frequency, or increased wavelength. The dispersion relations for the bandgaps of the final silicon structure are shown in figure 5. The speed of light line is drawn and it intersects large regions of gaps 5, 6, 7 and 8.

First, the defect was set to only occupy the first layer of capillaries (the one circle in the center) in the PBG fiber. A search for accelerating modes in the SOL intersection regions was done in BandSOLVE and then in CUDOS. One accelerating mode was found for this structure, and its longitudinal electric field ($E_z$) is shown for all points in the transverse (x-y) plane in figure 6. Next, the defect was set to occupy the first two layers of capillaries (the one circle in the center, and the six circles surrounding it) in the PBG fiber. Two accelerating modes were found for this structure, one in the “gap 5” bandgap, and one in the “gap 7” bandgap. Their longitudinal electric fields ($E_z$) are shown in figures 7 and 8, respectively. The electric field shown in figure 7 is more confined than the electric fields of the other two modes, and has less electromagnetic energy radiating out of the structure.

However, when crudely calculating the tolerances for these three modes, it was found that if some dimensions were perturbed by more than 1 nanometer, the mode would be lost completely. The modes were so sensitive, because they were at just the edge of the bandgap. Therefore, these modes were impractical, so the structure had to be modified to support
modes with higher tolerances. The presence of a defect tends to shift modes up in frequency, and the modes were shifted up too far, out of the defect, so the structure was modified to have a smaller defect, in order to lessen that effect.

Finally, the defect radius was modified to be slightly smaller, and a mode was found in the middle of bandgap 5. The longitudinal electric field $E_z$, shown in figure 9, was scaled for the material and plotted as a function of both $x$ and $y$ in figures 10 and 11, respectively. The accelerating gradient of this mode was found to be 618 MeV/m. While this is smaller than the accelerating gradient found for silica, it is still more than an order of magnitude better than the SLAC linac. The silicon structure is also good because the defect is much larger than defects in previously studied accelerating structures, providing a larger aperture for the electron beam, which should reduce additional damage that can come from the electron beam. Higher index of refraction materials may be the key to increasing the defect radius. In silica, when the defect radius is expanded to occupy the first two layers of capillaries, the electric field in the center falls to only a few percent of the maximum electric field in the glass. Here, in silicon, the electric field in the defect is approximately $1/3$ of the maximum electric field in the surrounding material, a remarkable improvement.

4.2.2 Sextapole Mode

An electric sextapole mode was also found in the silicon structure, in the same bandgap as the accelerating mode. The original CUDOS data was rotated $90^\circ$ to match the sextapole orientation normally used in beam physics.[17] The transverse fields $E_x$ and $E_y$ are shown in figures 12 and 13, respectively. In order to fit the fields to those expected from an ideal sextapole, the field distribution for an ideal sextapole had to first be calculated. The fields were only relevant in the defect, where the electron beam would pass, so we find the electric potential in that region. The defect is charge-free, so we consider solutions to Laplace’s equation in cylindrical coordinates,

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$  (13)
We assume that $\phi$ is separable, i.e. $\phi(r, \theta, z) = R(r)Q(\theta)Z(z)$ and that $\phi$ does not vary with $z$. Laplace’s equation then reduces to two separate equations,

$$\frac{r}{R} \frac{\partial}{\partial r} \left( r \frac{R}{\partial r} \right) = -\frac{1}{Q} \frac{\partial^2 Q}{\partial \theta^2} = n^2. \quad (14)$$

The right hand side gives $R_n(r) = A_n r^n + B_n r^{-n}$, and the left hand side gives $Q_n(\theta) = C_n \sin n\theta + D_n \cos n\theta$. We eliminate the $B_n r^{-n}$ term, because we require $E = 0$ at $r = 0$. Therefore, the general solution is,

$$\phi = \sum_{n=1}^{\infty} A_n r^n (C_n \cos n\theta + D_n \sin n\theta). \quad (15)$$

For the seaptapole we take the $n = 3$ mode,

$$\phi = A_3 r^3 (C_3 \cos 3\theta + D_3 \sin 3\theta) \quad (16)$$

$$= A_3 r^3 (\cos^3 \theta - 3 \cos \theta \sin \theta) + B_3 r^3 (3 \cos^2 \theta \sin \theta - \sin^3 \theta) \quad (17)$$

$$= A_3 (x^3 - 3xy^2) + B_3 (3x^2 y - y^3). \quad (18)$$

Now, we can find the electric field,

$$\vec{E} = \nabla \phi = A_3 ((3x^2 - 3y^2)\hat{x} - (6xy)\hat{y}) + B_3 ((6xy)\hat{x} + (3x^2 - 3y^2)\hat{y}) \quad [17].$$

In the mode that was found, $E_x = 0$ for $x = 0$ and $y = 0$, $E_y = 0$ for $y = x$ and $y = -x$, so $A_3 = 0$, and $\vec{E} = C((6xy)\hat{x} + (3x^2 - 3y^2)\hat{y})$. The CUDOS field data was plotted in MATLAB to see how well it matched with this theoretically predicted seaptapole mode. CUDOS only outputs the absolute value of the fields, so let $E_x$ and $E_y$ stand for the absolute values of the $x$ and $y$ components of the electric field. An electron beam would stay well confined to the center of the defect, so only the region within 80% of the defect radius was considered. In figure 14, $E_y$ vs. $x$ was plotted for $y = 0$, and in figure 15, $E_y$ vs. $y$ was plotted for $x = 0$. These were compared to the theoretical values along these lines $E_y = 3Cx^2$ and $E_y = 3Cy^2$, respectively. In figure 16 $E_x$ vs. $x$ was plotted for $y = x$, and in figure 17 $E_x$ vs. $x$ was plotted for $y = -x$. These were compared to the theoretical values along these lines $E_x = 6Cxy = 6x^2$ and $E_x = 6Cx(-y) = 6x^2$, respectively. Quadratic fits were done on the curves, and the $C$ values for the figures 14, 15, 16, and 17 were calculated to be 128, 84, 107, and 106 MV/m, respectively. These $C$ values are reasonably close to each other, so this mode is predominantly a seaptapole. Higher order polynomial fits on the curves were performed, and the quadratic term was always dominant by at least an order of magnitude.
5 Discussion and Conclusion

Photonic bandgap fiber accelerators can support accelerating gradients of as much as 5.55 GeV/m, which is two orders of magnitude better than the 25 MeV/m gradient of the SLAC linac. Technology that supports higher accelerating gradients is needed in order to continue probing higher energy regimes. Cost will eventually limit the size of accelerators. PBG fiber accelerators have the potential to be a much more effective and less expensive option, and could allow the next linear collider to be much shorter. Another advantage of PBG fiber accelerators are their built-in focusing capabilities. The sextapole mode was just one example.

The silicon structure has not yet been searched for all possible modes. Fabrication of the structures necessary for acceleration is a difficult problem, but progress has been made to bridge the gap between theoretical, ideal structures with the desired behavior and actual structures with tolerances. A systematic study of manufacturing errors and how they would effect the modes would be useful and has not yet been done. Also, the coupling of the laser light and the electron beam into the fiber is an open problem that the AARD is currently working on. Once a suitable structure can be fabricated, an experiment can be done where a real electron beam is accelerated and focused by modes excited in the structure. Incom is working on their manufacturing process and is achieving smaller and smaller microcapillary radii. Etching structures into silicon wafers is also a good possibility. Nanofabrication etching has a limited aspect ratio, but there is also the option of stacking multiple wafers to achieve the desired length. With further research and development, both technologies should be able to meet the scales necessary for particle acceleration. This study confirms that photonic bandgap fibers are a promising option for future particle accelerators and collaborations with fabrication facilities suggest that we will be able to manufacture them on a time scale of only a couple of years.
6 Acknowledgements

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References


7 Tables and Figures

Figure 1: Cross section of the RSoft CAD model of the Lin fiber, a PBG fiber with a central defect. The background is fused silica, and the red circles are holes.

<table>
<thead>
<tr>
<th>Fiber</th>
<th>Material</th>
<th>n</th>
<th>n_{eff}</th>
<th>λ (µm)</th>
<th>r (µm)</th>
<th>R (µm)</th>
<th>p (µm)</th>
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Table 1: PBG fibers studied. n is the index of refraction, n_{eff} is the effective index of refraction, λ is the wavelength of the accelerating mode, r is the radius of the outer cylinders, R is the defect radius, p is the pitch, or the center to center spacing of the outer cylinders, and N_{missing} is the number of rings removed to make the defect.
Figure 2: Magnitude of $E_z$ for the first accelerating mode in the IncomD fiber, shown at different points in the $x - y$ plane. The different colors depict the different field strengths, where blue is zero field, and red is the strongest field.

Figure 3: $E_z$ vs. $x$ for the first accelerating mode in the scaled IncomD fiber. The vertical lines show boundaries between the background material and the holes. $E_z$ inside the defect is constant, as desired.
Figure 4: $E_z$ vs. $y$ for the first accelerating mode in the scaled IncomD fiber. The vertical lines show boundaries between the background material and the holes. The electric field inside the defect is constant, as desired.

Figure 5: Dispersion relations for bandgaps in silicon fiber. The speed of light line is also drawn, and it intersects gaps 5, 6, 7 and 8.
Figure 6: $E_z$ for the only accelerating mode in the CIS-A fiber.

Figure 7: $E_z$ for the gap 5 defect mode in the CIS-B fibers.
Figure 8: $E_z$ for the gap 7 defect mode in the CIS-B fibers.

Figure 9: $E_z$ for the gap 5 defect mode in the CIS-final fibers.
Figure 10: $E_z$ vs. $x$ for the gap 5 defect mode in the CIS-final-2 fiber. $E_z$ is constant inside the defect, as desired.

Figure 11: $E_z$ vs. $y$ for the gap 5 defect mode in the CIS-final-2 fiber. $E_z$ is constant inside the defect, as desired.
Figure 12: $E_x$ for the sextapole mode in the CIS-final-2 fiber.

Figure 13: $E_y$ for the sextapole mode in the CIS-final-2 fiber.
Figure 14: $E_y$ vs. $x$ along the line $y = 0$ for the sextapole mode in the CIS-final-2 fiber. A quadratic fit was done on the data within 80% of the defect radius. The theoretically predicted curve is $E_y = 3Cx^2$, where $C$ is a constant defined above.

Figure 15: $E_y$ vs. $y$ along the line $x = 0$ for the sextapole mode in the CIS-final-2 fiber. A quadratic fit was done on the data within 80% of the defect radius. The theoretically predicted curve is $E_y = 3Cy^2$, where $C$ is a constant defined above.
Figure 16: $E_x$ vs. $x$ along the line $y = x$ for the sextapole mode in the CIS-final-2 fiber. A quadratic fit was done on the data within 80% of the defect radius. The theoretically predicted curve is $E_x = 6C x^2$, where $C$ is a constant defined above.

Figure 17: $E_x$ vs. $x$ along the line $y = -x$ for the sextapole mode in the CIS-final-2 fiber. A quadratic fit was done on the data within 80% of the defect radius. The theoretically predicted curve is $E_y = 6C x^2$, where $C$ is a constant defined above.